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Series in Condensed Matter Physics

Understanding Quantum Phase Transitions

Lincoln D. Carr

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Dedication

To Badia, Samuel, and Halim
For their patience and love
And to the three magical children
Who appeared in my life as I completed this book
Ahmed, Oumaima, and Yassmina

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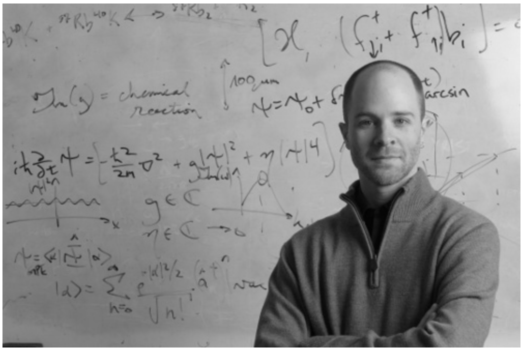

Lincoln D. Carr is a theoretical physicist who works primarily in quantum many-body theory, artificial materials, and nonlinear dynamics. He obtained his B.A. in physics at the University of California, Berkeley in 1994. He attended the University of Washington in Seattle from 1996 to 2001, where he received both his M.S. and Ph.D. in physics. He was a Distinguished International Fellow of the National Science Foundation from 2001-2004 at the Ecole normale supérieure in Paris and a professional research associate at JILA in Boulder, Colorado from 2003-2005. He joined the faculty in the physics department at the Colorado School of Mines in 2005, where he is presently an associate professor. He is an Associate of the National Institute of Standards and Technology and has been a visiting researcher at the Max Planck Institute for the Physics of Complex Systems in Dresden, Germany, the Kavli Institute of Theoretical Physics in Santa Barbara, California, the Institute Henri Poincaré at the Université Pierre et Marie Curie in Paris, and the Kirchhoff Institute for Physics at the University of Heidelberg.

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Preface

Phase transitions occur in all fields of the physical sciences and are crucial in engineering as well; abrupt changes from one state of matter to another are apparent everywhere we look, from the freezing of rivers to the steam rising up from the tea kettle. But why should it be only temperature and pressure that drive such abrupt transitions? In fact, quantum fluctuations can replace thermal fluctuations, a phase transition can occur even at zero temperature, and the concept of a phase transition turns out to be a lot more general than it is made out to be in elementary thermodynamics. Over the last twenty or so years the field of *quantum phase transitions* (QPTs) has seen steady growth. This book focuses especially on the latter half of this development. There are now so many experimental examples of QPTs that we hardly have space to include them all in a single volume. New numerical methods have opened up quantum many-body problems thought impossible to solve or understand. We can treat open and closed systems; we begin to understand the role of entanglement; we find or predict QPTs in naturally occurring systems ranging from chunks of matter to neutron stars, as well as engineered ones like quantum dots.

There are now almost five thousand papers devoted to QPTs. This book gives us a chance to pause and look back as well as to look forward to the future and the many open problems that remain. QPTs are a frontier area of research in many-body quantum mechanics, particularly in condensed matter physics. While we emphasize condensed matter, we include an explicit section at the end on QPTs across physics, and connections to other fields appear throughout the text. The book is divided into five parts, each containing from four to seven chapters.

Part I is intended to be somewhat more accessible to advanced graduate students and researchers entering the field. Thus it includes four more pedagogical, slightly longer chapters, covering new concepts and directions in QPTs: finite temperature and transport, dissipation, dynamics, and topological phases. Each of these chapters leads the reader from simpler ideas and concepts to the latest advances in these areas. The last two chapters of Part I cover entanglement, an important new tool for analysis of quantum many-body systems: first from a quantum-information-theoretic perspective, then from a geometrical picture tied to physical observables.

Part II delves into specific models and systems, in seven chapters. These are more closely tied to particular experimental realizations or theoretical methods. The topics include topological order, the Kondo lattice, ultracold

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quantum gases, dissipation and cavity quantum electrodynamics (QED), spin systems and group theory, Hubbard models, and metastability and finite-size effects.

Part III covers experiments, in six chapters. Although the book is mainly theoretical, the experimental chapters are key to making our whole discussion of QPTs meaningful; there are many observations now supporting the theories laid out in these pages. We present a selection covering a range of such experiments, including quantum dots, 2D electron systems, high-T_c materials, molecular systems, heavy fermions, and ultracold quantum gases in optical lattices.

Part IV presents recent advances in the key numerical methods used to study QPTS, in five chapters. These include the worm algorithm for quantum Monte Carlo, cluster Monte Carlo for dissipative QPTs, time-dependent density matrix renormalization group methods, new ideas in matrix product state methods, and dynamical mean field theory.

Finally, Part V presents a selection of QPTs in fields besides condensed matter physics, in four chapters. These include neutron stars and the quark-gluon plasma, cavity QED, nuclei, and a new mapping, now used by many string theorists, from classical gravitational theories (anti-de Sitter space) to conformal quantum field theories.

You can read this book by skipping around from topic to topic; that is how I edited it. However, in retrospect, I strongly recommend spending some time in Part I before delving into whichever topics catch your interest in the rest of the book. I also recommend reading thoroughly one or two experimental chapters early on in your perusing of this text, as it puts the rest in perspective.

This book tells its own story, and besides a few words of thanks, I won't delay you further with my remarks.

First and foremost, I thank the authors, who wrote amazing chapters from which I learned a tremendous amount. It is their writing that made the two years of effort I spent taking this book from conception to completion worth every last minute. The layout of the book and topic choices, although ultimately my own choice and my own responsibility, received useful input from many of the authors, for which I am also thankful.

I am grateful to the Aspen Center for Physics, which hosted a number of authors of this book, including myself, while we wrote our respective chapters. I am grateful to the Kirchhoff Institute for Physics and the Graduate School for Fundamental Physics at the University of Heidelberg, for hosting me during an important initial phase of the book.

I thank my post-doc and graduate students who offered a student perspective on these chapters, ensuring the text would be useful for physicists at levels ranging from graduate student to emeritus professor: Dr. Miguel-Ángel García-March, Laith Haddad, Dr. David Larue, Scott Strong, and Michael Wall. I thank Jim McNeil and Chip Durfee for their perspectives on nuclear physics and quantum optics, respectively, which they brought to bear in supplemental reviews for Part V, and Jim Bernard and David Wood for their

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