chapter three

Manufacturing cost analysis

Cost concepts and definitions

Cost management in a project environment refers to the functions required to maintain effective financial control of the project throughout its life cycle. There are several cost concepts that influence the economic aspects of managing projects. Within a given scope of analysis, there may be a combination of different types of cost aspects to consider. These cost aspects include the ones discussed below:

**Actual Cost of Work Performed.** This represents the cost actually incurred and recorded in accomplishing the work performed within a given time period.

**Applied Direct Cost.** This represents the amounts recognized in the time period associated with the consumption of labor, material, and other direct resources, without regard to the date of commitment or the date of payment. These amounts are to be charged to work-in-process (WIP) when resources are actually consumed, material resources are withdrawn from inventory for use, or material resources are received and scheduled for use within 60 days.

**Budgeted Cost for Work Performed.** This is the sum of the budgets for completed work plus the appropriate portion of the budgets for level of effort and apportioned effort. Apportioned effort is effort that by itself is not readily divisible into short-span work packages but is related in direct proportion to measured effort.

**Budgeted Cost for Work Scheduled.** This is the sum of budgets for all work packages and planning packages scheduled to be accomplished (including work-in-process), plus the amount of effort and apportioned effort scheduled to be accomplished within a given period of time.

**Direct Cost.** This is a cost that is directly associated with actual operations of a project. Typical sources of direct costs are direct material costs and direct labor costs. Direct costs are those that can be reasonably measured and allocated to a specific component of a project.

**Economies of Scale.** This refers to a reduction of the relative weight of the fixed cost in total cost by increasing output quantity. This helps
to reduce the final unit cost of a product. Economies of scale is often simply referred to as the savings due to mass production.

**Estimated Cost at Completion.** This is the actual direct cost, plus indirect costs that can be allocated to the contract, plus the estimate of costs (direct and indirect) for authorized work remaining.

**First Cost.** This is the total initial investment required to initiate a project or the total initial cost of the equipment needed to start the project.

**Fixed Cost.** This is a cost incurred irrespective of the level of operation of a project. Fixed costs do not vary in proportion to the quantity of output. Example of costs that make up the fixed cost of a project are administrative expenses, certain types of taxes, insurance cost, depreciation cost, and debt servicing cost.

**Incremental Cost.** This refers to the additional cost of changing the production output from one level to another. Incremental costs are normally variable costs.

**Indirect Cost.** This is a cost that is indirectly associated with project operations. Indirect costs are those that are difficult to assign to specific components of a project. An example of an indirect cost is the cost of computer hardware and software needed to manage project operations. Indirect costs are usually calculated as a percentage of a component of direct costs. For example, the direct costs in an organization may be computed as 10% of direct labor costs.

**Life-cycle Cost.** This is the sum of all costs, recurring and nonrecurring, associated with a project during its entire life cycle.

**Maintenance Cost.** This is a cost that occurs intermittently or periodically for the purpose of keeping project equipment in good operating condition.

**Marginal Cost.** This is the additional cost of increasing production output by one additional unit. The marginal cost is equal to the slope of the total cost curve or line at the current operating level.

**Operating Cost.** This is a recurring cost needed to keep a project in operation during its life cycle. Operating costs may consist of such items as labor cost, material cost, and energy cost.

**Opportunity Cost.** This is the cost of forgoing the opportunity to invest in a venture that would have produced an economic advantage. Opportunity costs are usually incurred due to limited resources that make it impossible to take advantage of all investment opportunities. It is often defined as the cost of the best rejected opportunity. Opportunity costs can be incurred due to a missed opportunity rather than due to an intentional rejection. In many cases, opportunity costs are hidden or implied because they typically relate to future events that cannot be accurately predicted.
Overhead Cost. This is a cost incurred for activities performed in support of the operations of a project. The activities that generate overhead costs support the project efforts rather than contribute directly to the project goal. The handling of overhead costs varies widely from company to company. Typical overhead items are electric power cost, insurance premiums, cost of security, and inventory carrying cost.

Standard Cost. This is a cost that represents the normal or expected cost of a unit of the output of an operation. Standard costs are established in advance. They are developed as a composite of several component costs such as direct labor cost per unit, material cost per unit, and allowable overhead charge per unit.

Sunk Cost. This is a cost that occurred in the past and cannot be recovered under the present analysis. Sunk costs should have no bearing on the prevailing economic analysis and project decisions. Ignoring sunk costs is always a difficult task for analysts. For example, if $950,000 was spent four years ago to buy a piece of equipment for a technology-based project, a decision on whether or not to replace the equipment now should not consider that initial cost. But uncompromising analysts might find it difficult to ignore that much money. Similarly, an individual planning on selling a personal automobile would typically try to relate the asking price to what was paid for the automobile when it was acquired. This is wrong under the strict concept of sunk costs.

Total Cost. This is the sum of all the variable and fixed costs associated with a project.

Variable Cost. This is a cost that varies in direct proportion to the level of operation or quantity of output. For example, the costs of material and labor required to make an item will be classified as variable costs because they vary with changes in level of output.

Cost and cash flow analysis
The basic reason for performing economic analysis is to make a choice between mutually exclusive projects that are competing for limited resources. The cost performance of each project will depend on the timing and levels of its expenditures. The techniques of computing cash flow equivalence permit us to bring competing project cash flows to a common basis for comparison. The common basis depends on the prevailing interest rate. Two cash flows that are equivalent at a given interest rate will not be equivalent at a different interest rate. The basic techniques for converting cash flows from one point in time to another are presented in the next section.
Time value of money calculations

Cash flow conversion involves the transfer of project funds from one point in time to another. The following notation is used for the variables involved in the conversion process:

\[ i = \text{interest rate per period} \]
\[ n = \text{number of interest periods} \]
\[ P = \text{a present sum of money} \]
\[ F = \text{a future sum of money} \]
\[ A = \text{a uniform end-of-period cash receipt or disbursement} \]
\[ G = \text{a uniform arithmetic gradient increase in period-by-period payments or disbursements}. \]

In many cases, the interest rate used in performing economic analysis is set equal to the minimum attractive rate of return (MARR) of the decision maker. The MARR is also sometimes referred to as hurdle rate, required internal rate of return (IRR), return on investment (ROI), or discount rate. The value of MARR is chosen with the objective of maximizing the economic performance of a project.

**Compound amount factor**
The procedure for the single payment compound amount factor finds a future sum of money, \( F \), that is equivalent to a present sum of money, \( P \), at a specified interest rate, \( i \), after \( n \) periods. This is calculated as

\[ F = P(1 + i)^n \]

**Example**  A sum of $5,000 is deposited in a project account and left there to earn interest for 15 years. If the interest rate per year is 12%, the compound amount after 15 years can be calculated as follows:

\[ F = 5000(1 + 0.12)^{15} \]
\[ = 27,367.85 \]

**Present worth factor**
The present worth factor computes \( P \) when \( F \) is given. The present worth factor is obtained by solving for \( P \) in the equation for the compound amount factor. That is,

\[ P = F(1 + i)^{-n} \]

Suppose it is estimated that $15,000 would be needed to complete the implementation of a project five years from now. How much should be
deposited in a special project fund now so that the fund would accrue to
the required $15,000 exactly five years from now? If the special project
fund pays interest at 9.2% per year, the required deposit would be

\[ P = 15000(1 + 0.092)^{-5} \]

\[ = 9660.03 \]

**Uniform series present worth factor**
The uniform series present worth factor is used to calculate the pres-
ent worth equivalent, \( P \), of a series of equal end-of-period amounts, \( A \). The derivation of the formula uses the finite sum of the present worth
of the individual amounts in the uniform series cash flow as shown
below.

\[ P = \sum_{i=1}^{n} A(1+i)^{-1} \]

\[ = A \left[ \frac{(1+i)^n - 1}{i(1+i)^n} \right] \]

**Example**
Suppose the sum of $12,000 must be withdrawn from an account to meet
the annual operating expenses of a multiyear project. The project account
pays interest at 7.5% per year compounded on an annual basis. If the proj-
ect is expected to last 10 years, how much must be deposited in the project
account now so that the operating expenses of $12,000 can be withdrawn
at the end of every year for 10 years? The project fund is expected to be
depleted to zero by the end of the last year of the project. The first with-
drawal will be made one year after the project account is opened, and no
additional deposits will be made in the account during the project life
cycle. The required deposit is calculated to be

\[ P = 12000 \left[ \frac{(1+0.075)^{10} - 1}{0.075(1+0.075)^{10}} \right] \]

\[ = 82,368.92 \]

**Uniform series capital recovery factor**
The capital recovery formula is used to calculate the uniform series of
equal end-of-period payments, \( A \), that are equivalent to a given present
amount, \( P \). This is the converse of the uniform series present amount fac-
tor. The equation for the uniform series capital recovery factor is obtained
by solving for \( A \) in the uniform series present amount factor. That is,
Example
Suppose a piece of equipment needed to launch a project must be purchased at a cost of $50,000. The entire cost is to be financed at 13.5% per year and repaid on a monthly installment schedule over four years. It is desired to calculate what the monthly loan payments will be. It is assumed that the first loan payment will be made exactly one month after the equipment is financed. If the interest rate of 13.5% per year is compounded monthly, then the interest rate per month will be 13.5%/12 = 1.125% per month. The number of interest periods over which the loan will be repaid is 4(12) = 48 months. Consequently, the monthly loan payments are calculated to be

\[
A = P \left[ \frac{i(1+i)^n}{(1+i)^n - 1} \right]
\]

\[
A = \$50,000 \left[ \frac{0.01125(1 + 0.01125)^{48}}{(1 + 0.01125)^{48} - 1} \right]
\]

\[
= \$1,353.82
\]

Uniform series compound amount factor
The series compound amount factor is used to calculate a single future amount that is equivalent to a uniform series of equal end-of-period payments. Note that the future amount occurs at the same point in time as the last amount in the uniform series of payments. The factor is derived as shown below:

\[
F = \sum_{i=1}^{n} A(1+i)^{n-1}
\]

\[
= A \left[ \frac{(1+i)^n - 1}{i} \right]
\]

Example
If equal end-of-year deposits of $5,000 are made to a project fund paying 8% per year for 10 years, how much can be expected to be available for withdrawal from the account for capital expenditure immediately after the last deposit is made?

\[
F = 5000 \left[ \frac{(1 + 0.08)^{10} - 1}{0.08} \right]
\]

\[
= \$72,432.50
\]
**Uniform series sinking fund factor**

The sinking fund factor is used to calculate the uniform series of equal end-of-period amounts, $A$, that are equivalent to a single future amount, $F$. This is the reverse of the uniform series compound amount factor. The formula for the sinking fund is obtained by solving for $A$ in the formula for the uniform series compound amount factor. That is,

$$A = F \left[ \frac{i}{(1+i)^n - 1} \right]$$

**Example**

How large are the end-of-year equal amounts that must be deposited into a project account so that a balance of $75,000 will be available for withdrawal immediately after the twelfth annual deposit is made? The initial balance in the account is zero at the beginning of the first year. The account pays 10% interest per year. Using the formula for the sinking fund factor, the required annual deposits are

$$A = 75,000 \left[ \frac{0.10}{(1+0.10)^{12} - 1} \right]$$

$$= 3,507.25$$

**Capitalized cost formula**

Capitalized cost refers to the present value of a single amount that is equivalent to a perpetual series of equal end-of-period payments. This is an extension of the series present worth factor with an infinitely large number of periods.

Using the limit theorem from calculus as $n$ approaches infinity, the series present worth factor reduces to the following formula for the capitalized cost:

$$P = \lim_{n \to \infty} A \left[ \frac{(1+i)^n - 1}{i(1+i)^n} \right]$$

$$= A \left\{ \lim_{n \to \infty} \left[ \frac{(1+i)^n - 1}{i(1+i)^n} \right] \right\}$$

$$= A \left( \frac{1}{i} \right)$$

**Example**

How much should be deposited in a general fund to service a recurring public service project to the tune of $6,500 per year forever if the fund
yields an annual interest rate of 11%? Using the capitalized cost formula, the required one-time deposit to the general fund is

\[ P = \frac{6500}{0.11} \]

\[ = $59,090.91 \]

The formulas presented above represent the basic cash flow conversion factors. The factors are widely tabulated, for convenience, in engineering economy books. Several variations and extensions of the factors are available. Such extensions include the arithmetic gradient series factor and the geometric series factor. Variations in the cash flow profiles include situations where payments are made at the beginning of each period rather than at the end and situations where a series of payments contains unequal amounts. Conversion formulas can be derived mathematically for those special cases by using the basic factors presented above.

**Arithmetic gradient series**

The gradient series cash flow involves an increase of a fixed amount in the cash flow at the end of each period. Thus, the amount at a given point in time is greater than the amount at the preceding period by a constant amount. This constant amount is denoted by \( G \). The size of the cash flow in the gradient series at the end of period \( t \) is calculated as

\[ A_i = (t - 1)G, \quad t = 1, 2, \ldots, n \]

The total present value of the gradient series is calculated by using the present amount factor to convert each individual amount from time \( t \) to time 0 at an interest rate of \( i \)% per period and summing up the resulting present values. The finite summation reduces to a closed form as shown below:

\[
P = \sum_{t=1}^{n} A_t(1+i)^{-t}
\]

\[
= \sum_{t=1}^{n} (t-1)G(1+i)^{-t}
\]

\[
= G \sum_{t=1}^{n} (t-1)(1+i)^{-t}
\]

\[
= G \left[ \frac{(1+i)^n - (1+ni)}{i^2(1+i)^n} \right]
\]
Example
The cost of supplies for a 10-year period increases by $1,500 every year starting at the end of year two. There is no supplies cost at the end of the first year. If interest rate is 8% per year, determine the present amount that must be set aside at time zero to take care of all the future supplies expenditures. We have \( G = 1500 \), \( i = 0.08 \), and \( n = 10 \). Using the arithmetic gradient formula, we obtain

\[
P = 1500 \left[ \frac{1 - (1 + 10(0.08))(1 + 0.08)^{-10}}{(0.08)^2} \right]
\]

\[
= 1500(25.9768)
\]

\[
= 38,965.20
\]

In many cases, an arithmetic gradient starts with some base amount at the end of the first period and then increases by a constant amount thereafter. The non-zero base amount is denoted as \( A_1 \).

The calculation of the present amount for such cash flows requires breaking the cash flow into a uniform series cash flow of amount \( A_1 \) and an arithmetic gradient cash flow with zero base amount. The uniform series present worth formula is used to calculate the present worth of the uniform series portion, while the basic gradient series formula is used to calculate the gradient portion. The overall present worth is then calculated as

\[
P = P_{\text{uniform series}} + P_{\text{gradient series}}
\]

\[
= A_1 \left[ \frac{(1 + i)^n - 1}{i(1 + i)^n} \right] + G \left[ \frac{(1 + i)^n - (1 + ni)}{i^2(1 + i)^n} \right]
\]

Increasing geometric series cash flow
In an increasing geometric series cash flow, the amounts in the cash flow increase by a constant percentage from period to period. There is a positive base amount, \( A_1 \), at the end of period 1. The amount at time \( t \) is denoted as

\[
A_t = A_{t-1}(1 + j), \quad t = 2, 3, \ldots, n
\]

where \( j \) is the percentage increase in the cash flow from period to period. By doing a series of back substitutions, we can represent \( A_t \) in terms of \( A_1 \) instead of in terms of \( A_{t-1} \) as shown below:

\[
A_2 = A_1(1 + j)
\]

\[
A_3 = A_2(1 + j) = A_1(1 + j)(1 + j)
\]

\[
\ldots
\]

\[
A_t = A_1(1 + j)^{t-1}, \quad t = 1, 2, 3, \ldots, n
\]
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The formula for calculating the present worth of the increasing geometric series cash flow is derived by summing the present values of the individual cash flow amounts. That is,

\[ P = \sum_{i=1}^{n} A_i (1+i)^{-t} \]

\[ = \sum_{i=1}^{n} [A_i (1+j)^{(i-1)}] (1+i)^{-t} \]

\[ = \frac{A_1}{(1+j)} \sum_{i=1}^{n} \left( \frac{1+j}{1+i} \right)^i \]

\[ = A_1 \left[ \frac{1-(1+j)^n (1+i)^{-n}}{i-j} \right], \quad i \neq j \]

If \( i = j \), the formula above reduces to the limit as \( i \to j \), shown below:

\[ P = \frac{nA_1}{1+i}, \quad i = j \]

**Example**

Suppose funding for a five-year project is to increase by 6% every year with an initial funding of $20,000 at the end of the first year. Determine how much must be deposited into a budget account at time zero in order to cover the anticipated funding levels if the budget account pays 10% interest per year. We have \( j = 6\% \), \( i = 10\% \), \( n = 5 \), \( A_1 = $20,000 \). Therefore,

\[ P = 20,000 \left[ \frac{1-(1+0.06)^5 (1+0.10)^{-5}}{0.10 - 0.06} \right] \]

\[ = 20,000(4.2267) \]

\[ = $84,533.60 \]

**Decreasing geometric series cash flow**

In a decreasing geometric series cash flow, the amounts in the cash flow decrease by a constant percentage from period to period. The cash flow starts at some positive base amount, \( A_1 \), at the end of period 1. The amount of time \( t \) is denoted as

\[ A_t = A_{t-1}(1-j), \quad t = 2, 3, \ldots, n \]
where \( j \) is the percentage decrease in the cash flow from period to period. As in the case of the increasing geometric series, we can represent \( A_t \) in terms of \( A_1 \):

\[
A_2 = A_1(1 - j) \\
A_3 = A_2(1 - j) = A_1(1 - j)(1 - j) \\
\ldots
\]

\[
A_t = A_1(1 - j)^{t-1}, \quad t = 1, 2, 3, \ldots, n
\]

The formula for calculating the present worth of the decreasing geometric series cash flow is derived by finite summation as in the case of the increasing geometric series. The final formula is

\[
P = A_1 \left[ \frac{1 - (1 - j)^n(1 + i)^{-n}}{i + j} \right]
\]

**Example**

The contract amount for a three-year project is expected to decrease by 10% every year with an initial contract of $100,000 at the end of the first year. Determine how much must be available in a contract reservoir fund at time zero in order to cover the contract amounts. The fund pays 10% interest per year. Because \( j = 10\% \), \( i = 10\% \), \( n = 3 \), and \( A_1 = $100,000 \), we should have

\[
P = 100,000 \left[ \frac{1 - (1 - 0.10)3(1 + 0.10)^{-3}}{0.10 + 0.10} \right]
\]

\[
= $100,000(2.2615)
\]

\[
= $226,150
\]

**Internal rate of return**

The internal rate of return (IRR) for a cash flow is defined as the interest rate that equates the future worth at time \( n \) or present worth at time 0 of the cash flow to zero. If we let \( i^* \) denote the internal rate of return, then we have

\[
\text{FW}_{t=n} = \sum_{t=0}^{n} (\pm A_t)(1 + i^*)^{n-t} = 0
\]

\[
\text{PW}_{t=0} = \sum_{t=0}^{n} (\pm A_t)(1 + i^*)^{-t} = 0
\]
where “+” is used in the summation for positive cash flow amounts or receipts and “—” is used for negative cash flow amounts or disbursements. $A_t$ denotes the cash flow amount at time $t$, which may be a receipt (+) or a disbursement (−). The value of $i^*$ is referred to as discounted cash flow rate of return, internal rate of return, or true rate of return. The procedure above essentially calculates the net future worth or the net present worth of the cash flow. That is,

$$\text{Net future worth} = \text{Future worth of receipts} - \text{Future worth of disbursements}$$

$$\text{NFW} = FW_{\text{receipts}} - FW_{\text{disbursements}}$$

$$\text{Net present worth} = \text{Present worth of receipts} - \text{Present worth of disbursements}$$

$$\text{NPW} = PW_{\text{receipts}} - PW_{\text{disbursements}}$$

Setting the NPW or NFW equal to zero and solving for the unknown variable $i$ determines the internal rate of return of the cash flow.

**Benefit-cost ratio**
The benefit-cost ratio of a cash flow is the ratio of the present worth of benefits to the present worth of costs. This is defined as

$$B / C = \frac{\sum_{t=0}^{n} B_t (1 + i)^{-t}}{\sum_{t=0}^{n} C_t (1 + i)^{-t}} = \frac{PW_{\text{benefits}}}{PW_{\text{costs}}}$$

where $B_t$ is the benefit (receipt) at time $t$ and $C_t$ is the cost (disbursement) at time $t$. If the benefit-cost ratio is greater than 1, then the investment is acceptable. If the ratio is less than 1, the investment is not acceptable. A ratio of 1 indicates a break-even situation for the project.

**Simple payback period**
Payback period refers to the length of time it will take to recover an initial investment. The approach does not consider the impact of the time value of money. Consequently, it is not an accurate method of evaluating the worth of an investment. However, it is a simple technique that is used widely to perform a “quick-and-dirty” assessment of investment.
performance. Also, the technique considers only the initial cost. Other costs that may occur after time zero are not included in the calculation. The payback period is defined as the smallest value of \( n (n_{\text{min}}) \) that satisfies the following expression:

\[
\sum_{t=1}^{n_{\text{min}}} R_t \geq C_0
\]

where \( R_t \) is the revenue at time \( t \) and \( C_0 \) is the initial investment. The procedure calls for a simple addition of the revenues period by period until enough total has been accumulated to offset the initial investment.

**Example**

An organization is considering installing a new computer system that will generate significant savings in material and labor requirements for order processing. The system has an initial cost of $50,000. It is expected to save the organization $20,000 a year. The system has an anticipated useful life of five years with a salvage value of $5,000. Determine how long it would take for the system to pay for itself from the savings it is expected to generate. Because the annual savings are uniform, we can calculate the payback period by simply dividing the initial cost by the annual savings. That is,

\[
n_{\text{min}} = \frac{50,000}{20,000} = 2.5 \text{ years}
\]

Note that the salvage value of $5,000 is not included in the above calculation because the amount is not realized until the end of the useful life of the asset (i.e., after five years). In some cases, it may be desired to consider the salvage value. In that case, the amount to be offset by the annual savings will be the net cost of the asset. In that case, we would have

\[
n_{\text{min}} = \frac{50,000 - 5000}{20,000} = 2.25 \text{ years}
\]

If there are tax liabilities associated with the annual savings, those liabilities must be deducted from the savings before calculating the payback period.

**Discounted payback period**

In this book, we introduce the *discounted payback period* approach in which the revenues are reinvested at a certain interest rate. The payback period
is determined when enough money has been accumulated at the given interest rate to offset the initial cost as well as other interim costs. In this case, the calculation is done by the following expression:

$$\sum_{t=1}^{n_{\text{min}}} R_t (1+i)^{n_{\text{min}}-t} \geq \sum_{t=0}^{n_{\text{min}}} C_t$$

**Example**

A new solar cell unit is to be installed in an office complex at an initial cost of $150,000. It is expected that the system will generate annual cost savings of $22,500 on the electricity bill. The solar cell unit will need to be overhauled every five years at a cost of $5,000 per overhaul. If the annual interest rate is 10%, find the discounted payback period for the solar cell unit considering the time value of money. The costs of overhaul are to be considered in calculating the discounted payback period.

Using the single payment compound amount factor for one period iteratively, the following solution is obtained:

Time period 1: $22,500
Time period 2: $22,500 + $22,500(1.10)^1 = $47,250
Time period 3: $22,500 + $47,250(1.10)^1 = $74,475
Time period 4: $22,500 + $74,475(1.10)^1 = $104,422.50
Time period 5: $22,500 + $104,422.50(1.10)^1 - $5000 = $132,364.75
Time period 6: $22,500 + $132,364.75(1.10)^1 = $168,101.23

The initial investment is $150,000. By the end of period 6, we have accumulated $168,101.23, more than the initial cost. Interpolating between period 5 and period 6, we obtain

$$n_{\text{min}} = 5 + \frac{150,000 - 132,364.75}{168,101.23 - 132,364.75} (6 - 5) = 5.49$$

That is, it will take 5.49 years, or five years and six months, to recover the initial investment.

**Investment life for multiple returns**

The time it takes an amount to reach a certain multiple of its initial level is often of interest in many investment scenarios. The “Rule of 72” is one simple approach to calculating how long it will take an investment to double in value at a given interest rate per period. The Rule of 72 gives the following formula for estimating the doubling period:

$$n_{\text{min}} = \frac{72}{i}$$
where \( i \) is the interest rate expressed in percentage. Referring to the single payment compound amount factor, we can set the future amount equal to twice the present amount and then solve for \( n \), the number of periods. That is, \( F = 2P \). Thus,

\[
2P = P(1 + i)^n
\]

Solving for \( n \) in the above equation yields an expression for calculating the exact number of periods required to double \( P \):

\[
n = \frac{\ln(2)}{\ln(1 + i)}
\]

where \( i \) is the interest rate expressed in decimals. In the general case, for exact computation, the length of time it would take to accumulate \( m \) multiple of \( P \) is expressed as

\[
n = \frac{\ln(m)}{\ln(1 + i)}
\]

where \( m \) is the desired multiple. For example, at an interest rate of 5% per year, the time it would take an amount, \( P \), to double in value \((m = 2)\) is 14.21 years. This, of course, assumes that the interest rate will remain constant throughout the planning horizon.

**Effects of inflation**

Inflation is a major player in financial and economic analyses of projects. Multiyear projects are particularly subject to the effects of inflation. Inflation can be defined as the decline in purchasing power of money.

Some of the most common causes of inflation are:

- Increase in amount of currency in circulation
- Shortage of consumer goods
- Escalation of the cost of production
- Arbitrary increase of prices by resellers

The general effects of inflation are felt in terms of increase in the prices of goods and decrease in the worth of currency. In cash flow analysis, return on investment (ROI) for a project will be affected by time value
of money as well as inflation. The real interest rate \((d)\) is defined as the desired rate of return in the absence of inflation. When we talk of “today’s dollars” or “constant dollars,” we are referring to the use of real interest rate. Combined interest rate \((i)\) is the rate of return combining real interest rate and inflation rate. If we denote the inflation rate as \(j\), then the relationship between the different rates can be expressed as

\[
1 + i = (1 + d)(1 + j)
\]

Thus, the combined interest rate can be expressed as

\[
i = d + j + dj
\]

Note that if \(j = 0\) (i.e., no inflation), then \(i = d\). We can also define commodity escalation rate \((g)\) as the rate at which individual commodity prices escalate. This may be greater than or less than the overall inflation rate. In practice, several measures are used to convey inflationary effects. Some of these are consumer price index, producer price index, and wholesale price index. A “market basket” rate is defined as the estimate of inflation based on a weighted average of the annual rates of change in the costs of a wide range of representative commodities. A “then-current” cash flow is a cash flow that explicitly incorporates the impact of inflation. A “constant worth” cash flow is a cash flow that does not incorporate the effect of inflation. The real interest rate, \(d\), is used for analyzing constant worth cash flows.

The then-current cash flow is the equivalent cash flow considering the effect of inflation. \(C_k\) is what it would take to buy a certain “basket” of goods after \(k\) time periods if there was no inflation. \(T_k\) is what it would take to buy the same “basket” in \(k\) time period if inflation was taken into account. For the constant worth cash flow, we have

\[
C_k = T_0, \quad k = 1, 2, \ldots, n
\]

and for the then-current cash flow, we have

\[
T_k = T_0 (1 + j)^k, \quad k = 1, 2, \ldots, n
\]

where \(j\) is the inflation rate. If \(C_k = T_0 = $100\) under the constant worth cash flow, then we mean $100 worth of buying power. If we are using the commodity escalation rate, \(g\), then we will have

\[
T_k = T_0 (1 + g)^k, \quad k = 1, 2, \ldots, n
\]
Thus, a then-current cash flow may increase based on both a regular inflation rate \((j)\) and a commodity escalation rate \((g)\). We can convert a then-current cash flow to a constant worth cash flow by using the following relationship:

\[
C_k = T_k (1 + j)^{-k}, \quad k = 1, 2, \ldots, n
\]

If we substitute \(T_k\) from the commodity escalation cash flow into the expression for \(C_k\) above, we get

\[
C_k = T_k (1 + j)^{-k} = T_0 (1 + g)^k (1 + j)^{-k} = T_0 [(1 + g) / (1 + j)]^k, \quad k = 1, 2, \ldots, n
\]

Note that if \(g = 0\) and \(j = 0\), the \(C_k = T_0\). That is, no inflationary effect. We now define effective commodity escalation rate \((v)\) as

\[
v = \left[\frac{(1 + g)}{(1 + j)}\right] - 1
\]

and we can express the commodity escalation rate \((g)\) as

\[
g = v + j + vj.
\]

Inflation can have a significant impact on the financial and economic aspects of a project. Inflation may be defined, in economic terms, as the increase in the amount of currency in circulation, resulting in a relatively high and sudden fall in its value. To a producer, inflation means a sudden increase in the cost of items that serve as inputs for the production process (equipment, labor, materials, etc.). To the retailer, inflation implies an imposed higher cost of finished products. To an ordinary citizen, inflation portends an unbearable escalation of prices of consumer goods. All these views are interrelated in a project management environment.

The amount of money supply, as a measure of a country’s wealth, is controlled by the government. With no other choice, governments often feel impelled to create more money or credit to take care of old debts and pay for social programs. When money is generated at a faster rate than the growth of goods and services, it becomes a surplus commodity, and its value (purchasing power) will fall. This means that there will be too much money available to buy only a few goods and services. When the purchasing power of a currency falls, each individual in a product’s life cycle has
to dispense more of the currency in order to obtain the product. Some of the classic concepts of inflation are discussed below:

1. Increases in producer’s costs are passed on to consumers. At each stage of the product’s journey from producer to consumer, prices are escalated disproportionately in order to make a good profit. The overall increase in the product’s price is directly proportional to the number of intermediaries it encounters on its way to the consumer. This type of inflation is called cost-driven (or cost-push) inflation.

2. Excessive spending power of consumers forces an upward trend in prices. This high spending power is usually achieved at the expense of savings. The law of supply and demand dictates that the more the demand, the higher the price. This type of inflation is known as demand-driven (or demand-pull) inflation.

3. Impact of international economic forces can induce inflation in a local economy. Trade imbalances and fluctuations in currency values are notable examples of international inflationary factors.

4. Increasing base wages of workers generates more disposable income and, hence, higher demands for goods and services. The high demand, consequently, creates a rise on prices. Coupled with this, employers pass on the additional wage cost to consumers through higher prices. This type of inflation is perhaps the most difficult to solve because wages set by union contracts and prices set by producers almost never fall – at least not permanently. This type of inflation may be referred to as wage-driven (or wage-push) inflation.

5. Easy availability of credit leads consumers to “buy now and pay later” and, thereby, creates another loophole for inflation. This is a dangerous type of inflation because the credit not only pushes prices up, but it also leaves consumers with less money later on to pay for the credit. Eventually, many credits become uncollectible debts, which may then drive the economy into recession.

6. Deficit spending results in an increase in money supply and, thereby, creates less room for each dollar to get around. The popular saying, “a dollar does not go far anymore,” simply refers to inflation in layman’s terms. The different levels of inflation may be categorized as discussed below.

**Mild inflation**

When inflation is mild (2%-4%), the economy actually prospers. Producers strive to produce at full capacity in order to take advantage of the high prices to the consumer. Private investments tend to be brisk and more jobs become available. However, the good fortune may only be temporary. Prompted by the prevailing success, employers are tempted to seek larger profits and
workers begin to ask for higher wages. They cite their employer’s prosperous business as a reason to bargain for bigger shares of the business profit. Thus, we end up with a vicious cycle where the producer asks for higher prices, the unions ask for higher wages, and inflation starts an upward trend.

**Moderate inflation**

Moderate inflation occurs when prices increase at 5%–9%. Consumers start purchasing more as an edge against inflation. They would rather spend their money now than watch it decline further in purchasing power. The increased market activity serves to fuel further inflation.

**Severe inflation**

Severe inflation is indicated by price escalations of 10% or more. Double-digit inflation implies that prices rise much faster than wages do. Debtors tend to be the ones who benefit from this level of inflation because they repay debts with money that is less valuable than the money borrowed.

**Hyperinflation**

When each price increase signals the increase in wages and costs, which again sends prices further up, the economy has reached a stage of malignant galloping inflation or hyperinflation. Rapid and uncontrollable inflation destroys the economy. The currency becomes economically useless as the government prints it excessively to pay for obligations.

Inflation can affect any project in terms of raw materials procurement, salaries and wages, and/or cost tracking dilemma. Some effects are immediate and easily observable. Other effects are subtle and pervasive. Whatever form it takes, inflation must be considered in long-term project planning and control. Large projects may be adversely affected by the effects of inflation in terms of cost overruns and poor resource utilization. The level of inflation will determine the severity of the impact on projects.

**Break-even analysis**

Break-even analysis refers to the determination of the balanced performance level where project income is equal to project expenditure. The total cost of an operation is expressed as the sum of the fixed and variable costs with respect to output quantity. That is,

\[
TC(x) = FC + VC(x)
\]

where \( x \) is the number of units produced, \( TC(x) \) is the total cost of producing \( x \) units, \( FC \) is the total fixed cost, and \( VC(x) \) is the total variable cost associated with producing \( x \) units. The total revenue resulting from the sale of \( x \) units is defined as
\[ TR(x) = px \]

where \( p \) is the price per unit. The profit due to the production and sale of \( x \) units of the product is calculated as

\[ P(x) = TR(x) - TC(x) \]

The break-even point of an operation is defined as the value of a given parameter that will result in neither profit nor loss. The parameter of interest may be the number of units produced, the number of hours of operation, the number of units of a resource type allocated, or any other measure of interest. At the break-even point, we have the following relationship:

\[ TR(x) = TC(x) \text{ or } P(x) = 0 \]

In some cases, there may be a known mathematical relationship between cost and the parameter of interest. For example, there may be a linear cost relationship between the total cost of a project and the number of units produced. The cost expressions facilitate straightforward break-even analysis. When two project alternatives are compared, the break-even point refers to the point of indifference between the two alternatives. The variable \( x_1 \) represents the point where projects A and B are equally desirable, \( x_2 \) represents where A and C are equally desirable, and \( x_3 \) represents where B and C are equally desirable. The analysis shows that if we are operating below a production level of \( x_2 \) units, then project C is the preferred project among the three. If we are operating at a level more than \( x_2 \) units, then project A is the best choice.

**Example**

Three project alternatives are being considered for producing a new product. The required analysis involves determining which alternative should be selected on the basis of how many units of the product are produced per year. Based on past records, there is a known relationship between the number of units produced per year, \( x \), and the net annual profit, \( P(x) \), from each alternative. The level of production is expected to be between 0 and 250 units per year. The net annual profits (in thousands of dollars) are given below for each alternative:

- Project A: \( P(x) = 3x - 200 \)
- Project B: \( P(x) = x \)
- Project C: \( P(x) = (1/50)x^2 - 300 \)
This problem can be solved mathematically by finding the intersection points of the profit functions and evaluating the respective profits over the given range of product units. It can also be solved by a graphical approach. Such a plot is called a *break-even chart*. A review of the calculations shows that Project B should be selected if between 0 and 100 units are to be produced. Project A should be selected if between 100 and 178.1 units (178 physical units) are to be produced. Project C should be selected if more than 178 units are to be produced. It should be noted that if less than 66.7 units (66 physical units) are produced, Project A will generate net loss rather than net profit. Similarly, Project C will generate losses if less than 122.5 units (122 physical units) are produced.

**Profit ratio analysis**

Break-even charts offer opportunities for several different types of analysis. In addition to the break-even points, other measures of worth or criterion may be derived from the charts. A measure, called *profit ratio* (Badiru and Omitaomu, 2007), is presented here for the purpose of obtaining a further comparative basis for competing projects. Profit ratio is defined as the ratio of the profit area to the sum of the profit and loss areas in a break-even chart. That is,

\[
\text{Profit ratio} = \frac{\text{Area of profit region}}{\text{Area of profit region} + \text{Area of loss region}}
\]

For example, suppose the expected revenue and the expected total cost associated with a project are given, respectively, by the following expressions:

\[
R(x) = 100 + 10x \\
TC(x) = 2.5x + 250
\]

where \(x\) is the number of units produced and sold from the project. The break-even point is shown to be 20 units. Net profits are realized from the project if more than 20 units are produced, and net losses are realized if less than 20 units are produced. It should be noted that the revenue function represents an unusual case where a revenue of $100 is realized when zero units are produced.

Suppose it is desired to calculate the profit ratio for this project if the number of units that can be produced is limited to between 0 and
100 units. The surface area of the profit region and the area of the loss region can be calculated by using the standard formula for finding the area of a triangle: Area = \((1/2)(\text{Base})(\text{Height})\). Using this formula, we have the following:

\[
\text{Area of profit region} = \frac{1}{2}(\text{Base})(\text{Height}) \\
= \frac{1}{2}(1100 - 500)(100 - 20) \\
= 24,000 \text{ square units}
\]

\[
\text{Area of loss region} = \frac{1}{2}(\text{Base})(\text{Height}) \\
= \frac{1}{2}(250 - 100)(20) \\
= 1500 \text{ square units}
\]

Thus, the profit ratio is computed as

\[
\text{Profit ratio} = \frac{24,000}{24,000 + 1500} \\
= 0.9411 \\
= 94.11\%
\]

The profit ratio may be used as a criterion for selecting among project alternatives. If this is done, the profit ratios for all the alternatives must be calculated over the same values of the independent variable. The project with the highest profit ratio will be selected as the desired project. Both the revenue and cost functions for the project are non-linear. The revenue and cost are defined as follows:

\[
R(x) = 160x - x^2 \\
TC(x) = 500 + x^2
\]

If the cost and/or revenue functions for a project are not linear, the areas bounded by the functions may not be easily determined. For those cases, it may be necessary to use techniques such as definite integrals to find the areas. The computations indicate that the project generates a loss if less than 3.3 units (3 actual units) or more than 76.8 (76 actual units) are produced. The respective profit and loss areas on the chart are calculated as follows:
Area 1 (loss) = \int_{0}^{3.3} \left[ (500 + x^2) - (160x - x^2) \right] dx \\
= 802.8 \text{ unit-dollars}

Area 2 (profit) = \int_{3.3}^{76.8} \left[ (160x - x^2) - (500 + x^2) \right] dx \\
= 132,272.08 \text{ unit-dollars}

Area 3 (loss) = \int_{76.8}^{100} \left[ (500 + x^2) - (160x - x^2) \right] dx \\
= 48,135.98 \text{ unit-dollars}

Consequently, the profit ratio for Project B is computed as

\[
\text{Profit ratio} = \frac{\text{Total area of profit region}}{\text{Total area of profit region} + \text{Total area of loss region}}
\]

\[
= \frac{132,272.08}{802.76 + 132,272.08 + 48,135.98}
\]

\[
= 0.7299
\]

\[
= 72.99\%
\]

The profit ratio approach evaluates the performance of each alternative over a specified range of operating levels. Most of the existing evaluation methods use single-point analysis with the assumption that the operating condition is fixed at a given production level. The profit ratio measure allows an analyst to evaluate the net yield of an alternative given that the production level may shift from one level to another. An alternative, for example, may operate at a loss for most of its early life, while it may generate large incomes to offset the losses in its later stages. Conventional methods cannot easily capture this type of transition from one performance level to another. In addition to being used to compare alternate projects, the profit ratio may also be used for evaluating the economic feasibility of a single project. In such a case, a decision rule may be developed. An example of such a decision rule is:

If profit ratio is greater than 75%, accept the project.
If profit ratio is less than or equal to 75%, reject the project.
Amortization analysis

Many capital investment projects are financed with external funds. A careful analysis must be conducted to ensure that the amortization schedule can be handled by the organization involved. A computer program such as GAMPS (graphic evaluation of amortization payments) might be used for this purpose (Badiru, 2016). The program analyzes the installment payments, the unpaid balance, principal amounts paid per period, total installment payment, and current cumulative equity. It also calculates the "equity break-even point" (Badiru, 2016) for the debt being analyzed. The equity break-even point indicates the time when the unpaid balance on a loan is equal to the cumulative equity on the loan. With the output of this program, the basic cost of servicing the project debt can be evaluated quickly. A part of the output of the program presents the percentage of the installment payment going into equity and interest charge respectively. The computational procedure for analyzing project debt follows the steps below:

1. Given a principal amount, \( P \), a periodic interest rate, \( i \) (in decimals), and a discrete time span of \( n \) periods, the uniform series of equal end-of-period payments needed to amortize \( P \) is computed as

\[
A = \frac{P[i(1+i)^n]}{(1+i)^n - 1}
\]

It is assumed that the loan is to be repaid in equal monthly payments. Thus, \( A(t) = A \), for each period \( t \) throughout the life of the loan.

2. The unpaid balance after making \( t \) installment payments is given by

\[
U(t) = \frac{A[1-(1+i)^{t-n}]}{i}
\]

3. The amount of equity or principal amount paid with installment payment number \( t \) is given by

\[
E(t) = A(1+i)^{t-n-1}
\]

4. The amount of interest charge contained in installment payment number \( t \) is derived to be

\[
I(t) = A[1-(1+i)^{t-n-1}]
\]

where \( A = E(t) + I(t) \).
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5. The cumulative total payment made after \( t \) periods is denoted by

\[
C(t) = \sum_{k=1}^{t} A(k)
\]

\[
= \sum_{k=1}^{t} A
\]

\[
= (A)(t)
\]

6. The cumulative interest payment after \( t \) periods is given by

\[
Q(t) = \sum_{x=1}^{t} I(x)
\]

7. The cumulative principal payment after \( t \) periods is computed as

\[
S(t) = \sum_{k=1}^{t} E(k)
\]

\[
= A \sum_{k=1}^{t} (1 + i)^{-(n-k+1)}
\]

\[
= A \left[ \frac{(1 + i)^t - 1}{i(1 + i)^n} \right]
\]

where:

\[
\sum_{n=1}^{t} x^n = \frac{x^{t+1} - x}{x - 1}
\]

8. The percentage of interest charge contained in installment payment number \( t \) is

\[
f(t) = \frac{I(t)}{A}(100\%)
\]

9. The percentage of cumulative interest charge contained in the cumulative total payment up to and including payment number \( t \) is

\[
F(t) = \frac{Q(t)}{C(t)}(100\%)
\]
10. The percentage of cumulative principal payment contained in the cumulative total payment up to and including payment number $t$ is

$$H(t) = \frac{S(t)}{C(t)}$$

$$= \frac{C(t) - Q(t)}{C(t)}$$

$$= 1 - \frac{Q(t)}{C(t)}$$

$$= 1 - F(t)$$

**Example**

Suppose that a manufacturing productivity improvement project is to be financed by borrowing $500,000 from an industrial development bank. The annual nominal interest rate for the loan is 10%. The loan is to be repaid in equal monthly installments over a period of 15 years. The first payment on the loan is to be made exactly one month after financing is approved. It is desired to perform a detailed analysis of the loan schedule.

The tabulated result shows a monthly payment of $5,373.04 on the loan. Considering time $t = 10$ months, one can see the following results:

- $U(10) = $487,475.13 (unpaid balance)
- $A(10) = $5373.04 (monthly payment)
- $E(10) = $1299.91 (equity portion of the tenth payment)
- $I(10) = $4073.13 (interest portion of the tenth payment)
- $C(10) = $53,730.40 (total payment to date)
- $S(10) = $12,526.21 (total equity to date)
- $f(10) = 75.81\%$ (percentage of the tenth payment going into interest charge)
- $F(10) = 76.69\%$ (percentage of the total payment going into interest charge)

Thus, over 76\% of the sum of the first 10 installment payments goes into interest charges. The analysis shows that by time $t = 180$, the unpaid balance has been reduced to zero. That is, $U(180) = 0.0$. The total payment made on the loan is $967,148.40 and the total interest charge is $967,148.20 - $500,000 = $467,148.20. Thus, 48.30\% of the total payment goes into interest charges. The information about interest charges might be very useful for tax purposes. The tabulated output, if used, would show that equity builds up
slowly while unpaid balance decreases slowly. Note that very little equity is accumulated during the first three years of the loan schedule. The effects of inflation, depreciation, property appreciation, and other economic factors are not included in the analysis presented above. A project analyst should include such factors whenever they are relevant to the loan situation.

The point at which the curves intersect is referred to as the *equity break-even point*. It indicates when the unpaid balance is exactly equal to the accumulated equity or the cumulative principal payment. For the example, the equity break-even point is 120.9 months (over 10 years). The importance of the equity break-even point is that any equity accumulated after that point represents the amount of ownership or equity that the debtor is entitled to after the unpaid balance on the loan is settled with project collateral. The implication of this is very important, particularly in the case of mortgage loans. “Mortgage” is a word with French origin, meaning *death pledge* – perhaps a sarcastic reference to the burden of mortgage loans. The equity break-even point can be calculated directly from the formula derived below:

Let the equity break-even point, $x$, be defined as the point where $U(x) = S(x)$. That is,

$$A \left[ \frac{1 - (1 + i)^{(n-x)}}{i} \right] = A \left[ \frac{(1 + i)^x - 1}{i(1+i)^n} \right]$$

Multiplying both the numerator and denominator of the left-hand side of the above expression by $(1 + i)^n$ and simplifying yields

$$\frac{(1 + i)^n - (1 + i)^x}{i(1+i)^n}$$

on the left-hand side. Consequently, we have

$$(1 + i)^n - (1 + i)^x = (1 + i)^x - 1$$

$$(1 + i)^x = \frac{(1 + i)^n + 1}{2}$$

which yields the equity break-even expression:

$$x = \frac{\ln[0.5(1+i)^n + 0.5]}{\ln(1+i)}$$

where:

- $\ln$ is the natural log function
- $n$ is the number of periods in the life of the loan
- $i$ is the interest rate per period.
The total payment starts from $0.0 at time zero and goes up to $967,147.20 by the end of the last month of the installment payments. Because only $500,000 was borrowed, the total interest payment on the loan is $967,147.20 − $500,000 = $467,147.20. The cumulative principal payment starts at $0.0 at time zero and slowly builds up to $500,001.34, which is the original loan amount. The extra $1.34 is due to round-off error in the calculations.

The percentage of interest charge in the monthly payments starts at 77.55% for the first month and decreases to 0.83% for the last month. By comparison, the percentage of interest in the total payment starts also at 77.55% for the first month and slowly decreases to 48.30% by the time the last payment is made at time 180. It is noted that an increasing proportion of the monthly payment goes into the principal payment as time goes on. If the interest charges are tax deductible, the decreasing values of $f(t)$ mean that there would be decreasing tax benefits from the interest charges in the later months of the loan.

Manufacturing cost estimation

Cost estimation and budgeting help establish a strategy for allocating resources in project planning and control. There are three major categories of cost estimation for budgeting. These are based on the desired level of accuracy. The categories are order-of-magnitude estimates, preliminary cost estimates, and detailed cost estimates. Order-of-magnitude cost estimates are usually gross estimates based on the experience and judgment of the estimator. They are sometimes called “ballpark” figures. These estimates are typically made without a formal evaluation of the details involved in the project. The level of accuracy associated with order-of-magnitude estimates can range from −50% to +50% of the actual cost. These estimates provide a quick way of getting cost information during the initial stages of a project.

\[
50\% (\text{Actual cost}) \leq \text{Order-of-magnitude estimate} \leq 150\% (\text{Actual cost})
\]

Preliminary cost estimates are also gross estimates, but with a higher level of accuracy. In developing preliminary cost estimates, more attention is paid to some selected details of the project. An example of a preliminary cost estimate is the estimation of expected labor cost. Preliminary estimates are useful for evaluating project alternatives before final commitments are made. The level of accuracy associated with preliminary estimates can range from −20% to +20% of the actual cost.

\[
80\% (\text{Actual cost}) \leq \text{Preliminary estimate} \leq 120\% (\text{Actual cost})
\]
Detailed cost estimates are developed after careful consideration is given to all the major details of a project. Because of the considerable amount of time and effort needed to develop detailed cost estimates, the estimates are usually developed after there is firm commitment that the project will take off. Detailed cost estimates are important for evaluating actual cost performance during the project. The level of accuracy associated with detailed estimates normally range from −5% to +5% of the actual cost.

\[ 95\% \text{ (Actual cost)} \leq \text{Detailed cost} \leq 105\% \text{ (Actual cost)} \]

There are two basic approaches to generating cost estimates. The first one is a variant approach, in which cost estimates are based on variations of previous cost records. The other approach is the generative cost estimation, in which cost estimates are developed from scratch without taking previous cost records into consideration.

**Optimistic and Pessimistic Cost Estimates.** Using an adaptation of the Program Evaluation and Review Technique (PERT) formula, we can combine optimistic and pessimistic cost estimates. Let:

- \( O \) = optimistic cost estimate
- \( M \) = most likely cost estimate
- \( P \) = pessimistic cost estimate

Then, the cost can be estimated as

\[
E[C] = \frac{O + 4M + P}{6}
\]

and the cost variance can be estimated as

\[
V[C] = \left( \frac{P - O}{6} \right)^2
\]

**Budgeting and capital allocation**

Budgeting involves sharing limited resources between several project groups or functions in a project environment. Budget analysis can serve any of the following purposes:

- A plan for resources expenditure
- A project selection criterion
- A projection of project policy
- A basis for project control
- A performance measure
- A standardization of resource allocation
- An incentive for improvement

**Top-down budgeting**

Top-down budgeting involves collecting data from upper-level sources such as top and middle managers. The numbers supplied by the managers may come from their personal judgment, past experience, or past data on similar project activities. The cost estimates are passed to lower-level managers, who then break the estimates down into specific work components within the project. These estimates may, in turn, be given to line managers, supervisors, and lead workers to continue the process until individual activity costs are obtained. Top management provides the global budget, while the functional level worker provides specific budget requirements for the project items.

**Bottom-up budgeting**

In this method, elemental activities, and their schedules, descriptions, and labor skill requirements are used to construct detailed budget requests. Line workers familiar with specific activities are requested to provide cost estimates. Estimates are made for each activity in terms of labor time, materials, and machine time. The estimates are then converted to an appropriate cost basis. The dollar estimates are combined into composite budgets at each successive level up the budgeting hierarchy. If estimate discrepancies develop, they can be resolved through the intervention of senior management, middle management, functional managers, project manager, accountants, or standard cost consultants.

Elemental budgets may be developed on the basis of the times progress of each part of the project. When all the individual estimates are gathered, we obtain a composite budget estimate. Analytical tools such as learning curve analysis, work sampling, and statistical estimation may be employed in the cost estimation and budgeting processes.

**Mathematical formulation of capital allocation**

Capital rationing involves selecting a combination of projects that will optimize the return on investment. A mathematical formulation of the capital budgeting problem is presented below:
Maximize $z = \sum_{i=1}^{n} v_i x_i$

Subject to $\sum_{i=1}^{n} c_i x_i \leq B$

$x_i = 0,1; \ i = 1,\ldots,n$

where:

$n =$ number of projects

$v_i =$ measure of performance for project $i$ (e.g., present value)

c_i =$ cost of project $i$

$x_i =$ indicator variable for project $i$

$B =$ budget availability level

A solution of the above model will indicate which projects should be selected in combination with which projects. The example that follows illustrates a capital rationing problem.

**Example**

Planning of portfolio of projects is essential in resource limited projects. The capital rationing example presented here (Badiru and Pulat, 1995) involves the determination of the optimal combination of project investments so as to maximize total return on investment. Suppose that a project analyst is given $N$ projects, $X_1, X_2, X_3, \ldots, X_N$, with the requirement to determine the level of investment in each project so that total investment return is maximized subject to a specified limit on available budget. The projects are not mutually exclusive.

The investment in each project starts at a base level $b_i (i=1, 2, \ldots, N)$ and increases by variable increments $k_{ij} (j=1, 2, 3, \ldots, K_i)$, where $K_i$ is the number of increments used for project $i$. Consequently, the level of investment in project $X_i$ is defined as

$$x_i = b_i + \sum_{j=1}^{K_i} k_{ij}$$

where:

$$x_i \geq 0 \ \ \ \forall i$$

For most cases, the base investment will be zero. In those cases, we will have $b_i=0$. In the modeling procedure used for this problem, we have
The variable $x_i$ is the actual level of investment in project $i$, while $X_i$ is an indicator variable indicating whether or not project $i$ is one of the projects selected for investment. Similarly, $k_{ij}$ is the actual magnitude of the $j$th increment while $Y_{ij}$ is an indicator variable that indicates whether or not the $j$th increment is used for project $i$. The maximum possible investment in each project is defined as $M_i$, such that

$$b_i \leq x_i \leq M_i$$

There is a specified limit, $B$, on the total budget available to invest such that

$$\sum_i x_i \leq B$$

There is a known relationship between the level of investment, $x_i$, in each project and the expected return, $R(x_i)$. This relationship will be referred to as the utility function, $f(\cdot)$, for the project. The utility function may be developed through historical data, regression analysis, and forecasting models. For a given project, the utility function is used to determine the expected return, $R(x_i)$, for a specified level of investment in that project. That is,

$$R(x_i) = f(x_i) = \sum_{j=1}^{k_i} r_{ij} Y_{ij}$$

where $r_{ij}$ is the incremental return obtained when the investment in project $i$ is increased by $k_{ij}$. If the incremental return decreases as the level of investment increases, the utility function will be concave. In that case, we will have the following relationship:

$$r_{ij} \geq r_{ij} + 1 \quad \text{or} \quad r_{ij} - r_{ij} + 1 \geq 0$$

Thus,

$$Y_{ij} \geq Y_{ij+1} \quad \text{or} \quad Y_{ij} - Y_{ij+1} \geq 0$$
so that only the first $n$ increments ($j=1, 2, \ldots, n$) that produce the highest returns are used for project $i$.

If the incremental returns do not define a concave function, $f(x_i)$, then one has to introduce the inequality constraints presented above into the optimization model. Otherwise, the inequality constraints may be left out of the model, since the first inequality, $Y_{ij} \geq Y_{ij+1}$, is always implicitly satisfied for concave functions. Our objective is to maximize the total return. That is,

$$\text{Maximize } Z = \sum_i \sum_j r_{ij} Y_{ij}$$

Subject to the following constraints:

$$x_i = b_i + \sum_j k_{ij} Y_{ij} \quad \forall i$$

$$b_i \leq x_i \leq M_i \quad \forall i$$

$$Y_{ij} \geq Y_{ij+1} \quad \forall i, j$$

$$\sum_i x_i \leq B$$

$$x_i \geq 0 \quad \forall i$$

$$Y_{ij} = 0 \text{ or } 1 \quad \forall i, j$$

Now suppose we are given four projects (i.e., $N=4$) and a budget limit of $10$ million.

For example, if an incremental investment of $0.20$ million from stage 2 to stage 3 is made in project 1, the expected incremental return from the project will be $0.30$ million. Thus, a total investment of $1.20$ million in project 1 will yield a total return of $1.90$ million.

The question addressed by the optimization model is to determine how many investment increments should be used for each project. That is, when should we stop increasing the investments in a given project? Obviously, for a single project, we would continue to invest as long as the incremental returns are larger than the incremental investments. However, for multiple projects, investment interactions complicate the decision so that investment in one project cannot be independent of the other projects. The LP model of the capital rationing example was solved with LINDO software. The solution indicates the following values for $Y_{ij}$.

**Project 1:**

$$Y_{11} = 1, \quad Y_{12} = 1, \quad Y_{13} = 1, \quad Y_{14} = 0, \quad Y_{15} = 0$$
Thus, the investment in project 1 is $X_1 = 1.20$ million. The corresponding return is $1.90$ million.

**Project 2:**

\[ Y_{21} = 1, \quad Y_{22} = 1, \quad Y_{23} = 1, \quad Y_{24} = 1, \quad Y_{25} = 0, \quad Y_{26} = 0, \quad Y_{27} = 0 \]

Thus, the investment in project 2 is $X_2 = 3.80$ million. The corresponding return is $6.80$ million.

**Project 3:**

\[ Y_{31} = 1, \quad Y_{32} = 1, \quad Y_{33} = 1, \quad Y_{34} = 1, \quad Y_{35} = 0, \quad Y_{36} = 0, \quad Y_{37} = 0 \]

Thus, the investment in project 3 is $X_3 = 2.60$ million. The corresponding return is $5.90$ million.

**Project 4:**

\[ Y_{41} = 1, \quad Y_{42} = 1, \quad Y_{43} = 1 \]

Thus, the investment in project 4 is $X_4 = 2.35$ million. The corresponding return is $3.70$ million.

The total investment in all four projects is $9,950,000$. Thus, the optimal solution indicates that not all of the $10,000,000$ available should be invested. The expected return from the total investment is $18,300,000$. This translates into 83.92% return on investment.

The optimal solution indicates an unusually large return on total investment. In a practical setting, expectations may need to be scaled down to fit the realities of the project environment. Not all optimization results will be directly applicable to real situations. Possible extensions of the above model of capital rationing include the incorporations of risk and time value of money into the solution procedure. Risk analysis would be relevant, particularly for cases where the levels of returns for the various levels of investment are not known with certainty. The incorporation of time value of money would be useful if the investment analysis is to be performed for a given planning horizon. For example, we might need to make investment decisions to cover the next five years rather than just the current time.

**Cost monitoring**

As a project progresses, costs can be monitored and evaluated to identify areas of unacceptable cost performance. A plot of cost versus time for projected cost and actual cost can reveal a quick identification of when cost overruns occur in a project.
Plots similar to those presented above may be used to evaluate cost, schedule, and time performance of a project. An approach similar to the profit ratio presented earlier may be used together with the plot to evaluate the overall cost performance of a project over a specified planning horizon. Presented below is a formula for cost performance index (CPI):

$$\text{CPI} = \frac{\text{Area of cost benefit}}{\text{Area of cost benefit} + \text{Area of cost overrun}}$$

As in the case of the profit ratio, CPI may be used to evaluate the relative performance of several project alternatives or to evaluate the feasibility and acceptability of an individual alternative.

**Project balance technique**

One other approach to monitoring cost performance is the project balance technique. The technique helps in assessing the economic state of a project at a desired point in time in the life cycle of the project. It calculates the net cash flow of a project up to a given point in time. The project balance is calculated as

$$B(i)_t = S_t - P(1+i)^t + \sum_{k=1}^{t} \text{PW}_{\text{income}}(i)_k$$

where:

- $B(i)_t$ = project balance at time $t$ at an interest rate of $i\%$ per period
- $\text{PW}_{\text{income}}(i)_t$ = present worth of net income from the project up to time $t$
- $P$ = initial cost of the project
- $S_t$ = salvage value at time $t$

The project balanced at time $t$ gives the net loss or net project associated with the project up to that time.

**Cost control system**

Contract management involves the process by which goods and services are acquired, utilized, monitored, and controlled in a project. Contract management addresses the contractual relationships from the initiation of a project to the completion of the project (i.e., completion of services and/or handover of deliverables). Some of the important aspects of contract management are
In 1967, the US Department of Defense (DoD) introduced a set of 35 standards or criteria which contractors must comply with under cost or incentive contracts. Although it is no longer actively required by DoD, the concepts and techniques behind the criteria are still very much applicable and useful for effective management of project costs. Further, the criteria formed the basis of more recent cost-management techniques. The system of criteria is referred to as the cost and schedule control systems criteria (C/SCSC). Many government agencies now require compliance with C/SCSC for major contracts. The purpose is to manage the risk of cost overrun to the government. The system presents an integrated approach to cost and schedule management. It is now widely recognized and used in major project environments. It is intended to facilitate greater uniformity and provide advance warning about impending schedule or cost overruns.

The topics covered by C/SCSC include cost estimating and forecasting, budgeting, cost control, cost reporting, earned value analysis, resource allocation and management, and schedule adjustments. The important link between all of these is the dynamism of the relationship between performance, time, and cost. This is essentially a multi-objective problem. Because performance, time, and cost objectives cannot be satisfied equally well, concessions or compromises would need to be worked out in implementing C/SCSC.

Another dimension of the performance–time–cost relationship is the US Air Force’s R&M 2000 standard which addresses reliability and maintainability of systems. R&M 2000 is intended to integrate reliability and maintainability into the performance, cost, and schedule management for government contracts. C/SCSC and R&M 2000 constitute an effective guide for project design. To comply with C/SCSC, contractors must use standardized planning and control methods based on earned value. Earned value refers to the actual dollar value of work performed at a given point in time compared to planned cost for the work.

This is different from the conventional approach of measuring actual versus planned, which is explicitly forbidden by C/SCSC. In the conventional approach, it is possible to misrepresent the actual content (or value) of the work accomplished. The work-rate analysis technique presented in
another section can be useful in overcoming the deficiencies of the conve-
tventional approach. C/SCSC is developed on a work content basis using
the following factors:

- The actual cost of work performed (ACWP), which is determined on
  the basis of the data from cost accounting and information systems
- The budgeted cost of work scheduled (BCWS) or baseline cost deter-
  mined by the costs of scheduled accomplishments
- The budgeted cost of work performed (BCWP) or earned value, the
  actual work of effort completed as of a specific point in time

The following equations can be used to calculate cost and schedule
variances for work packages at any point in time.

\[
\text{Cost variance} = \text{BCWP} - \text{ACWP} \\
\text{Percent cost variance} = \left( \frac{\text{Cost variance}}{\text{BCWP}} \right) \times 100 \\
\text{Schedule variance} = \text{BCWP} - \text{BCWS} \\
\text{Percent schedule variance} = \left( \frac{\text{Schedule variance}}{\text{BCWS}} \right) \times 100 \\
\text{ACWP and remaining funds} = \text{Target cost (TC)} \\
\text{ACWP} + \text{cost to complete} = \text{Estimated cost at completion (EAC)}
\]

Sources of capital

Financing a project means raising capital for the project. Capital is a
resource consisting of funds available to execute a project. Capital includes
not only privately owned production facilities but also public investment.
Public investments provide the infrastructure of the economy such as
roads, bridges, water supply, and so on. Other public capital that indi-
directly supports production and private enterprise include schools, police
stations, central financial institutions, and postal facilities.

If the physical infrastructure of the economy is lacking, the incentive
for private entrepreneurs to invest in production facilities is likely to be
lacking also. The government or community leaders can create the atmo-
sphere for free enterprise by constructing better roads, providing better
public safety, facilities, and encouraging ventures that assure adequate
support services.

As far as project investment is concerned, what can be achieved with
project capital is very important. The avenues for raising capital funds
include banks, government loans or grants, business partners, cash
reserves, and other financial institutions. The key to the success of the free
Manufacturing and enterprise system is the availability of capital funds and the availability of sources to invest the funds in ventures that yield products needed by the society. Some specific ways that funds can be made available for business investments are discussed below:

**Commercial Loans.** Commercial loans are the most common sources of project capital. Banks should be encouraged to loan money to entrepreneurs, particularly those just starting business. Government guarantees may be provided to make it easier for the enterprise to obtain the needed funds.

**Bonds and Stocks.** Bonds and stocks are also common sources of capital. National policies regarding the issuance of bonds and stocks can be developed to target specific project types to encourage entrepreneurs.

**Interpersonal Loans.** Interpersonal loans are unofficial means of raising capital. In some cases, there may be individuals with enough personal funds to provide personal loans to aspiring entrepreneurs. But presently, there is no official mechanism that handles the supervision of interpersonal business loans. If a supervisory body exists at a national level, wealthy citizens will be less apprehensive about loaning money to friends and relatives for business purposes. Thus, the wealthy citizens can become a strong source of business capital.

**Foreign Investment.** Foreign investments can be attracted for local enterprises through government incentives. The incentives may be in terms of attractive zoning permits, foreign exchange permits, or tax breaks.

**Investment Banks.** The operations of investment banks are often established to raise capital for specific projects. Investment banks buy securities from enterprises and resell them to other investors. Proceeds from these investments may serve as a source of business capital.

**Mutual Funds.** Mutual funds represent collective funds from a group of individuals. The collective funds are often large enough to provide capitals for business investments. Mutual funds may be established by individuals or under the sponsorship of a government agency. Encouragement and support should be provided for the group to spend the money for business investment purposes.

**Supporting Resources.** A clearing house of potential goods and services that a new project can provide may be established by the government. New entrepreneurs interested in providing the goods and services should be encouraged to start relevant enterprises. They should be given access to technical, financial, and information resources to facilitate starting production operations. As an example, the state of Oklahoma, under the auspices of the Oklahoma Center for the
Advancement of Science and Technology (OCAST), has established a resource database system. The system, named TRAC (Technical Resource Access Center), provides information about resources and services available to entrepreneurs in Oklahoma. The system is linked to the statewide economic development information system. This is a clearing house arrangement that will facilitate access to resources for project management.

The time value of money is an important factor in project planning and control. This is particularly crucial for long-term projects that are subject to changes in several cost parameters. Both the timing and quantity of cash flows are important for project management. The evaluation of a project alternative requires consideration of the initial investment, depreciation, taxes, inflation, economic life of the project, salvage value, and cash flows.

Activity-based costing

Activity-based costing (ABC) has emerged as an appealing costing technique in industry. The major motivation for ABC is that it offers an improved method to achieve enhancements in operational and strategic decisions. Activity-based costing offers a mechanism to allocate costs in direct proportion to the activities that are actually performed. This is an improvement over the traditional way of generically allocating costs to departments. It also improves the conventional approaches to allocating overhead costs.

The use of PERT/CPM, precedence diagramming, and the recently developed approach of critical resource diagramming (Badiru and Pulat, 1995) can facilitate task decomposition to provide information for activity-based costing. Some of the potential impacts of ABC on a production line are:

- Identification and removal of unnecessary costs
- Identification of the cost impact of adding specific attributes to a product
- Indication of the incremental cost of improved quality
- Identification of the value-added points in a production process
- Inclusion of specific inventory carrying costs
- Provision of a basis for comparing production alternatives
- Ability to assess “what-if” scenarios for specific tasks

Activity-based costing is just one component of the overall activity-based management in an organization. Activity-based management involves a more global management approach to planning and control
of organizational endeavors. This requires consideration for product planning, resource allocation, productivity management, quality control, training, line balancing, value analysis, and a host of other organizational responsibilities. Thus, while activity-based costing is important, one must not lose sight of the universality of the environment in which it is expected to operate. And, frankly, there are some processes where functions are so intermingled that decomposition into specific activities may be difficult. Major considerations in the implementation of ABC are:

- Resources committed to developing activity-based information and cost
- Duration and level of effort needed to achieve ABC objectives
- Level of cost accuracy that can be achieved by ABC
- Ability to track activities based on ABC requirements
- Handling the volume of detailed information provided by ABC
- Sensitivity of the ABC system to changes in activity configuration

Income analysis can be enhanced by activity-based costing approach. Similarly, instead of allocating manufacturing overhead on the basis of direct labor costs, an activity-based costing analysis could be performed. The specific ABC cost components can be further broken down if needed. A spreadsheet analysis would indicate the impact on net profit as specific cost elements are manipulated.

Conclusion

In any manufacturing enterprise, computations and analyses similar to those illustrated in this chapter are crucial in controlling and enhancing the bottom-line survival of the organization. Analysts can adapt and extend the techniques presented in this chapter for application to the prevailing scenarios in their respective organizations.

References