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Some Reflections on the History of Fluid Dynamics

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2.1 Introduction

As you read these words, there are millions of modern engineering devices in operation that depend in part, or in total, on the understanding of fluid dynamics—airplanes in flight, ships at sea, automobiles on the road, mechanical biomedical devices, etc. In the modern world, we sometimes take these devices for granted. However, it is important to pause for a moment and realize that each of these machines is a miracle of modern engineering fluid dynamics wherein many diverse fundamental laws of nature are harnessed and combined in a useful fashion so as to produce a safe, efficient, and effective machine. Indeed, the sight of an airplane flying overhead typifies the laws of aerodynamics in action, and it is easy to forget that just two centuries ago, these laws were so mysterious or unknown or misunderstood as to preclude a flying machine from even lifting off the ground, let alone successfully fly through the air.

In turn, this raises the question as to just how did our intellectual understanding of fluid dynamics evolve? To find the answer, we have to reach back over millennia of intellectual thought, all the way back to science in ancient Greece. However, to properly address the history of fluid dynamics in a complete fashion requires many more pages than available in this chapter. Several books have been written on the subject, notably those by Rouse and Ince (1957) and Tokaty (1971). An inclusive study of the history of both fluid dynamics and aerodynamics can be found in a recent book by Anderson (1997).

Instead, in this chapter we will focus on a few themes and case histories that exemplify the historical evolution of fluid

dynamics and provide a flavor of the intellectual thought and the human dynamics that have led to the state-of-the-art of fluid dynamics as we know it today. We will choose a chronological approach to the subject, and will cobble together advancements in both theoretical and experimental fluid dynamics. Much of the following material is excerpted from the author's broader study of the subject as found in Anderson (1997).

2.2 From the Greeks to Leonardo da Vinci

The science of fluid dynamics can trace its roots to a man born in 384 BC in the Ionian colony of Stagira in the Aegean Sea, and educated at Plato's Academy in Athens. Aristotle (384–322 BC) lived at the most intellectually fruitful time in Greek history, went to the best school, and associated with some of the most influential people. Through all of this, Aristotle developed a corpus of philosophy, science, ethics, and law that influenced the world for the following 2000 years.

Aristotle's scientific thoughts established two concepts that bear on the development of fluid dynamics. The first is the concept of a *continuum*. He wrote that

The continuous may be defined as that which is divisible into parts which are themselves divisible to infinity, as a body which is divisible in all ways. Magnitude divisible in one direction is a line, in three directions a body. And magnitudes which are divisible in this fashion are continuous.

It is not widely appreciated that the fundamental concept of a continuum, upon which most fluid dynamic theory is based, is one of Aristotle's contributions to the science of fluid dynamics.

The second of Aristotle's contributions to aerodynamics was the idea that a moving body passing through the air or another fluid encounters some aerodynamic "resistance." He wrote:

It is impossible to say why a body that has been set in motion in a vacuum should ever come to rest. Why, indeed, should it come to rest at one place rather than another. As a consequence, it will either necessarily stay at rest, or if in motion, will move indefinitely unless some obstacle comes into collision with it.

A conclusion from this reasoning is that, since bodies eventually come to rest in a fluid, there must be a *resistance* acting on the body. Today, we call this fluid dynamic *drag*.

The other ancient Greek scientist to contribute to fluid dynamics was born in 287 BC in Syracuse, and was killed unceremoniously in 212 BC by Roman soldiers while he was drawing geometric figures in the Syracuse sand. Archimedes is usually known for his concepts in fluid statics, and particularly for his vague concept of pressure in a fluid. He sensed that every point of the wetted surface area of a body in a fluid was under some force due to the fluid (although the concept of "force" was not quantified during the age of Greek science). However, there was some vague, intuitive feeling about what we today technically label as force, and Archimedes realized that such force is distributed over the body surface. Archimedes stated that, in a fluid, "each part is always pressed by the whole weight of the column perpendicularly above it." This was the first statement of the principle that, in modern terms, the pressure at a point in a stationary fluid is due to the weight of the fluid above it, and hence is linearly proportional to the depth of the fluid. This is a true statement, as long as the fluid is not in motion, that is, for fluid *statics*.

However, Archimedes made a contribution to fundamental fluid dynamics as follows. Today, we fully understand that, in order to set a stagnant fluid into motion, a *difference* in pressure must be exerted across the fluid. We call this pressure difference over a unit length the *pressure gradient*. Archimedes had a vague understanding of this point when he wrote, "if fluid parts are continuous and uniformly distributed, then that of them which is the least compressed is driven along by that which is more compressed." Liberally interpreted, this means that when a pressure gradient is imposed across a stagnant fluid, the fluid will start to move in the direction of decreasing pressure. The above statement by Archimedes is a clear contribution by Greek science to fluid dynamics.

In summary, examining the work of both Aristotle and Archimedes, we can find no conscious effort on the part of either person to study any major aspect of fluid dynamics. In a historical perspective, we would not expect otherwise. However, the fundamental thoughts of these men were the beginnings, no matter how obscure, of the fundamental science of fluid dynamics.

The time-span from the death of Archimedes to the time of Leonardo da Vinci covers the zenith of the Roman Empire, its fall, the dearth of intellectual activity in Western Europe during the Dark Ages, and the surge of new thought that characterized the Renaissance. In terms of the science of aerodynamics, the seventeen centuries that separate Archimedes and Leonardo resulted in no worthwhile contributions. Although the Romans excelled in highly organized civil, military, and political activities, as well as in large engineering feats with building construction and the wide distribution of water from reservoirs to cities via aqueducts, they contributed nothing of substance to scientific theory. Moreover, although the ancient Greek science and philosophy was kept alive for future generations by eastern Arabian cultures through the Dark Ages, no new contributions were made during this period.

This changed with the work of Leonardo da Vinci. Born in 1452 in the small Tuscan village of Vinci, near Florence, Leonardo da Vinci went on to revolutionize the worlds of art, science, and technology. He is recognized today as being in the forefront of the world's greatest intelligences.

Pertinent to this chapter, Leonardo had an interest in the characteristics of basic fluid flow. For example, one of the fundamental principles of modern fluid mechanics is the fact that mass is conserved; in terms of a fluid moving steadily in a tube, this means that the mass flow (e.g., the number of pounds per second) passing through any cross section of the tube is the same. For an incompressible flow (flow of a fluid, or low-speed flow of a gas), this principle leads to a basic relation that

$$AV = \text{constant}$$

where A is the cross-sectional area of the duct at any location, and V is the velocity of the fluid at that same location. This relation is called the *continuity equation*, and it states that in moving from one location in the duct to another where the area is smaller, the velocity becomes larger in just the right amount that the product of A times V remains the same. Leonardo observed and recorded this effect in regard to the flow of water in rivers, where, in those locations where the river becomes constricted, the water velocity increases. Moreover, he quantified this observation in the following statement with the accompanying sketch shown in Figure 2.1:

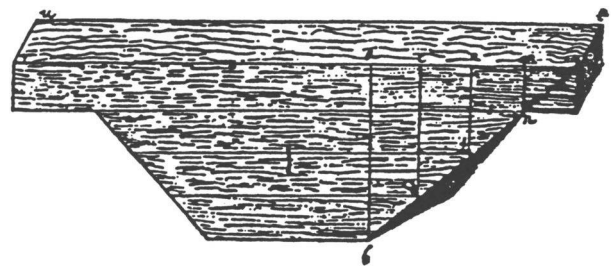


FIGURE 2.1 Flow in a variable area channel. (Sketch by Leonardo da Vinci.)

Each movement of water of equal surface width will run the swifter the smaller the depth ... and this motion will be of this quality: I say that in mn the water has more rapid movement than in ab , and as many times more as mn enters into ab ; it enters 4 times, the motion will therefore be 4 times as rapid in mn as in ab .

Here we have, for the first time in history, a quantitative statement of the special form of the continuity equation that holds for low-speed flow.

In addition to this quantitative contribution, Leonardo, being a consummate observer of nature, made many sketches of various flowfields. A particularly graphic example is shown in Figure 2.2, found in the *Codex Atlanticus*. Here we see the vortex structure of the flow around a flat plate. At the top, the plate is perpendicular to the flow, and Leonardo accurately sketches the recirculating, separated flow at the back of the plate, along with the extensive wake that trails downstream. At the bottom, the plate is aligned with the flow, and we see the vortex that is created at the junction of the plate surface and the water surface, as well as the bow wave that propagates at an angle away from the plate surface. These sketches by Leonardo are virtually identical to photographs of such flows that can be taken in any modern fluid dynamic laboratory, and they demonstrate the detail to which Leonardo observed various flow patterns.

In modern fluid dynamics and aerodynamics, the wind tunnel is an absolutely essential laboratory device. Although we take for granted today that the relative flow over a stationary body mounted in a wind tunnel is the same as the relative flow over the same body moving through a stationary fluid, we have Leonardo to thank for being the first to state this fact. His statement of what we can call today the “wind tunnel principle” can be found in two different parts of the *Codex Atlanticus*. Leonardo made

the following statements: “As it is to move the object against the motionless air so it is to move the air against the motionless object,” and “The same force as is made by the thing against air, is made by air against the thing.”

Therefore, the basic principle that allows us to make wind tunnel measurements and apply them to atmospheric flight was first conceived by Leonardo. Giacomelli (1920) has called this the “principle of aerodynamic reciprocity.”

2.3 The Velocity-Squared Law

We now address what is perhaps the most important breakthrough in experimental fluid dynamics in the seventeenth century. Put yourself in the shoes of a self-styled natural philosopher in the middle ages. In thinking about the question of how the force on an object immersed in a moving fluid varies with the velocity of the fluid, intuition is most likely to tell you that, when the velocity doubles, the force doubles. That is, you are inclined to feel that force is directly proportional to velocity. This seems “logical,” although there is (up to the seventeenth century) no proper experimental evidence or theoretical analysis to say one way or another. Like so much of ancient science, this feeling was based simply on the image of geometric perfection in nature, and what could be more “perfect” than the force doubling when the velocity doubles. Indeed, both Leonardo and Galileo—two of the greatest minds in history—held this belief. Up to the middle of the seventeenth century, the prevailing thought was the incorrect notion that force was directly proportional to the flow velocity.

However, within the space of 17 years at the end of the seventeenth century, this situation changed dramatically. Between 1673 and 1690, two independent sets of experiments due to Edme Mariotte (1620–1684) in France and Christian Huygens (1629–1695) in Holland, along with the theoretical fundamentals published by Isaac Newton (1642–1727) in England, clearly established that the force on an object varies as the *square* of the flow velocity; i.e., if the velocity doubles, the force goes up by a factor of four. In comparison to the previous centuries of halting, minimal progress in fluid dynamics, the rather sudden realization of the velocity-squared law for aerodynamic force represents the first *major* scientific breakthrough in the historical evolution of the subject. Let us examine this breakthrough more closely, as well as the men who made it possible.

Credit for the origin of the velocity-squared law rests with Edme Mariotte, who first published it in the year 1673. To gain an appreciation for the circumstances surrounding this development, let us consider Mariotte’s background. He lived in absolute obscurity for about the first 40 years of his life. There is even controversy as to where and when he was born. There is a claim that he was born in Dijon, France, in 1620, but there are no documents to verify this, let alone to pinpoint an exact birthdate. We have no evidence concerning his personal life, his education, or his vocation until 1666, when very suddenly he is made a charter member of the newly formed Paris Academy of Sciences. Most likely, Mariotte was self-taught in the sciences. He came to the attention of the Academy through his pioneering theory that sap

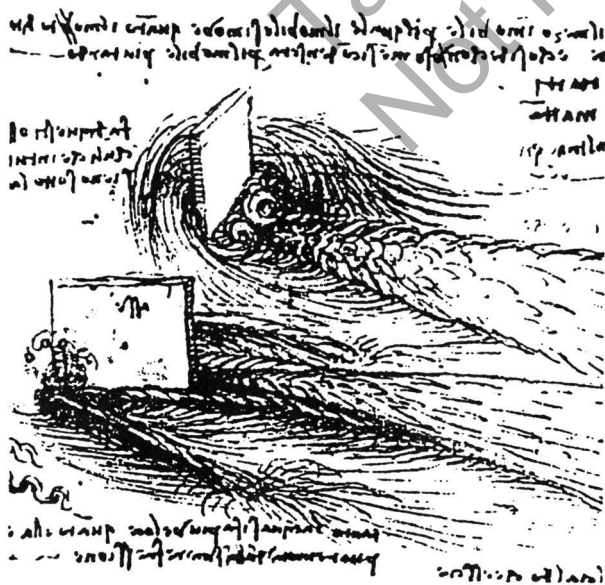


FIGURE 2.2 Flow around a flat plate. (Sketch by Leonardo da Vinci.)

circulated through plants in a manner analogous to blood circulating through animals. Controversial at that time, his theory was confirmed within 4 years by numerous experimental investigators. It is known that he was residing in Dijon at the time of his appointment to the Academy. Mariotte quickly proved to be an active member and contributor to the Academy. His areas of work were diverse; he was interested in experimental physics, hydraulics, optics, plant physiology, surveying, and general scientific and mathematical methodology. Mariotte is credited as the first in France to develop *experimental science*, transferring to that country the same interest in experiments that grew during the Italian renaissance with the work of Leonardo and Galileo. Indeed, Mariotte was a gifted experimenter who took pains to try to link existing theory to experiment—a novel thought in that day. The Academy was essentially Mariotte's later life; he remained in Paris until his death on May 12, 1684.

The particular work of Mariotte of interest to our discussion was conducted in the period before 1673. He was particularly interested in the forces produced by various bodies impacting on other bodies or surfaces. One of these "bodies" was a fluid; Mariotte examined and measured the force created by a moving fluid impacting on a flat surface. The device he used for these experiments is shown in Figure 2.3. Here we see a beam dynamometer wherein a stream of water impinges on one end of the beam, and the force exerted by this stream is balanced and measured by a weight on the other end of the beam. The water jet emanates from the bottom of a filled vertical tube, and its velocity is known from Torricelli's Law as a function of the height of the column of water in the tube. From the results obtained with this experimental apparatus, Mariotte was able to prove that the force

of impact of the water on the beam varied as the square of the flow velocity. He presented these results in a paper read to the Paris Academy of Science in 1673, entitled "Traite de la Percussion ou choc des Corp,"—the first time in history that the velocity-squared law was published. For this work, Edme Mariotte deserves the credit for the first major advancement toward the understanding of velocity effects on aerodynamic force.

As a final note on Mariotte, the esteem in which he was held by some of his colleagues is reflected by the words of J. B. du Hamel, who said after Mariotte's death in 1684,

The mind of this man was highly capable of all learning, and the works published by him attest to the highest erudition. In 1667, on the strength of a singular doctrine, he was elected to the Academy. In him, sharp inventiveness always shone forth combined with the industry to carry through, as the works referred to in the course of this treatise will testify. His cleverness in the design of experiments was almost incredible, and he carried them out with minimal expense.

However, there was at least one colleague who was not so happy with Mariotte, and who represents another side of the historical proprietorship of the velocity-squared law. This man was Christian Huygens (1629–1695). Indeed, in the 1930s Giacomelli and Pistolesi (1934) gave Huygens (not Mariotte) credit for the first proof of the velocity-squared law. Let us examine this situation further.

To begin with, Huygen's background is better known than that of Mariotte. Christian Huygens was born on April 14, 1629, in the Hague, Netherlands, to a family prominent in Dutch society. His grandfather served William the Silent and Prince Maurice as secretary. His father, Constantijn, was secretary to Prince Frederick Henry. Indeed, several members of his family were diplomats under the reign of the Orange family in Holland. Christian was well-educated; he was tutored by his father until the age of 16, after which he studied law and mathematics at the University of Leiden. Devoting himself to physics and mathematics, Huygens made substantial contributions, including improvements in existing methodology, developing new techniques in optics, and inventing the pendulum clock. Even today, all textbooks on basic physics discuss Huygens' law of optics. For his accomplishments, Huygens was made a charter member of the Paris Academy of Science in 1666—the same year as Mariotte. Huygens moved to Paris in order to more closely participate in the activities of the Academy; he lived in Paris until 1681. During this period, both Mariotte and Huygens worked, conversed, and argued together as colleagues in the Academy. In 1681, Huygens moved back to the Hague, where he died on July 8, 1695. During his life, Huygens was recognized as Europe's greatest mathematician. However, he was a somewhat solitary person who did not attract a following of young students. Moreover, he was reluctant to publish, mainly because of his inordinately high personal standards. For these two reasons, Huygens' work did not greatly influence the scientists of the next century; indeed, he became relatively unknown during the eighteenth century.

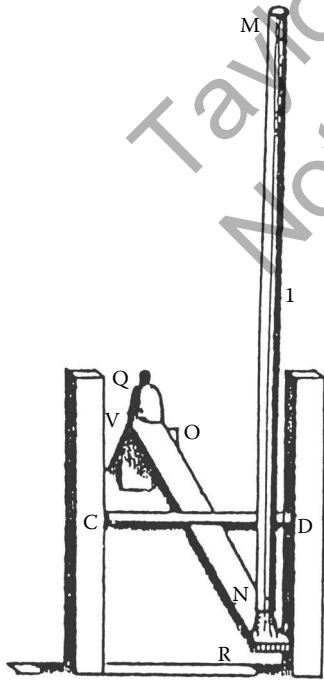


FIGURE 2.3 Beam dynamometer for measuring fluid force on an object used by Edme Mariotte.

In 1668, Huygens began to study the fall of projectiles in resisting media. Following Leonardo and Galileo, he started out with the belief that resistance (drag) was proportional to velocity. However, within 1 year his analysis of the experimental data convinced him that resistance was proportional to the square of the velocity. This was 4 years before Mariotte published the same result in 1673; however, Huygens delayed until 1690 in publishing his data and conclusions. This somewhat complicates the question as to whom should the velocity-squared law be attributed. The picture is further blurred by Huygens himself, who accused Mariotte of plagiarism; however, Huygens levied this charge *after* Mariotte's death in 1684. Huygens stated that "Mariotte took everything from me." In regard to Mariotte's paper in 1673, Huygens complains that "he should have mentioned me. I told him that one day, and he could not respond."

According to author's opinion, here is a classic situation that frequently occurs in scientific and engineering circles even in modern times. We have a learned society—the Paris Academy of Sciences—the members of which frequently gathered to discuss their experiments, theories, and general feelings about the natural world. Ideas and preliminary results were shared and critiqued in a collegial atmosphere. Mariotte and Huygens were colleagues, and from Huygens own words above, they clearly discussed and shared thoughts. In such an atmosphere, the exact credit for the origin of new ideas is sometimes not clear; ideas frequently evolve as a result of discussion among groups. What is clear is this. Mariotte published the velocity-squared law in a paper given to the Academy in 1673; Huygens published the same conclusion 17 years later. Moreover, in 1673 Huygens critiqued Mariotte's paper, and said *nothing* about plagiarism or not being referenced. Why did he wait until after Mariotte's death 11 years later to make such charges? This author has no definite answer to this question. However, using the written scientific literature as the measure of proprietorship, Mariotte is clearly the first person to publish the velocity-squared law. Taken in conjunction with Huygens' silence at the time of this publication, we have to conclude that Mariotte deserves first credit for this law. However, it is quite clear that Huygen's experiments, which were carried out before Mariotte's publication, also proved the velocity-squared law. Of course, of great importance to the development of fluid dynamics is simply the fact that, by the end of the seventeenth century, we have direct experimental proof from two independent investigations that fluid dynamics force varies as the square of the velocity. Of even greater importance is that, at the same time, the same law was derived *theoretically* on the basis of the rational, mathematical laws of mechanics advanced by Newton in his *Principia*, published in 1687.

2.4 The Sunrise of Hydrodynamics: Daniel Bernoulli and the Pressure–Velocity Concept

The fundamental advances in fluid dynamics that occurred in the eighteenth century began with the work of Daniel

Bernoulli (1700–1782). Newtonian mechanics had unlocked the door to modern hydrodynamics, but the door was still closed at the beginning of the century. Daniel Bernoulli was the first to open this door, albeit just by a crack; Euler and others who followed flung the door wide open.

Daniel Bernoulli was born in Groningen, Netherlands, on February 8, 1700. His father, Johann, was a professor at Groningen but returned to Basel, Switzerland, in 1705 to occupy the chair of mathematics that had been vacated by the death of Jacob Bernoulli. At the University of Basel, Daniel obtained a master's degree in 1716 in philosophy and logic. He went on to study medicine in Basel, Heidelberg, and Strasbourg, obtaining his PhD in anatomy and botany in 1721. During these studies, he maintained an active interest in mathematics. He followed this interest by moving briefly to Venice, where he published an important work titled *Exercitationes Mathematicae* in 1724. This earned him much attention and resulted in his winning the prize awarded by the Paris Academy—the first of 10 he was eventually to receive. In 1725, Daniel moved to St. Petersburg, Russia, to join the academy. The St. Petersburg Academy had gained a substantial reputation for scholarship and intellectual accomplishment at that time. During the next 8 years, Bernoulli experienced his most creative period. While at St. Petersburg, he wrote his famous book *Hydrodynamica*, completed in 1734, but not published until 1738. In 1733, Daniel returned to Basel to occupy the chair of anatomy and botany, and in 1750 moved to the chair of physics created exclusively for him. He continued to write, give very popular and well-attended lectures in physics, and make contributions to mathematics and physics until his death in Basel on March 17, 1782.

Daniel Bernoulli was famous in his own time. He was a member of virtually all the existing learned societies and academies, such as Bologna, St. Petersburg, Berlin, Paris, London, Bern, Turin, Zurich, and Mannheim. His importance to fluid dynamics is centered on his 1738 book, *Hydrodynamica* (with this book, Daniel introduced the term "hydrodynamics" to literature). In this book, he ranged over such topics as jet propulsion, manometers, and flow in pipes. Of most importance, however, he attempted to find a relationship between the variations of pressure with velocity in a fluid flow. He used Newtonian mechanics, along with the concept of "*vis viva*," or "living force" introduced by Leibniz in 1695. This was actually an energy concept; "*vis viva*" was defined by Leibniz as the product of mass times velocity squared, mV^2 ; today, we recognize this as twice the kinetic energy of a moving object of mass m . Also, Bernoulli treated pressure in terms of the height of a fluid, much as Archimedes had done 20 centuries previously; the concept that pressure is a *point* property that can vary from one point to another in a flow cannot be found in Bernoulli's work.

Let us critically examine Bernoulli's contribution to fluid dynamics. In modern fluid dynamics there exists the "Bernoulli principle," which simply states that *in a flowing fluid, as the velocity increases, the pressure decreases*. This is an absolute fact that is frequently used to explain the generation of lift on an airplane wing; as the flow speeds up while moving over the top surface of

the wing, the pressure decreases, and in turn this lower pressure exerts a “suction” on the top of the wing, thus generating lift. A quantitative statement of the Bernoulli principle is *Bernoulli’s equation*, written as follows. If points 1 and 2 are two different points in a fluid flow, then

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$$

This is the famous Bernoulli equation—perhaps the most famous equation in all of fluid dynamics. Examining this equation, clearly if V_2 is larger than V_1 , then p_2 is smaller than p_1 ; that is, as V increases, p decreases. Question: How much of this did Bernoulli ever state? The answer is, not much. In his book *Hydrodynamica*, which is the central reference used by all subsequent investigators for his contributions, Bernoulli did attempt to derive the relation between pressure and velocity. Using the concept of “*vis viva*” Bernoulli applied an energy conservation principle to the sketch shown in Figure 2.4; this is a copy of his original illustration for *Hydrodynamica*. Here, we see a large tank, ABGC, filled with water, to which has been attached a horizontal pipe, EFDG. The end of the pipe is partially closed; it contains a small orifice through which the water escapes. Stating that the sum of the potential and kinetic energies of the fluid in a pipe is constant (an incorrect statement, because in a flowing fluid there is work done by the pressure in addition to the existence of kinetic and potential energies—such “flow work” was not understood by Bernoulli), he obtained the following differential equation for the change in velocity, dV , over a small distance, dx ,

$$\frac{VdV}{dx} = \frac{a - V^2}{2c}$$

where a is the height of the water in the tank, and c is the length of the horizontal pipe. The above equation is a far cry from the

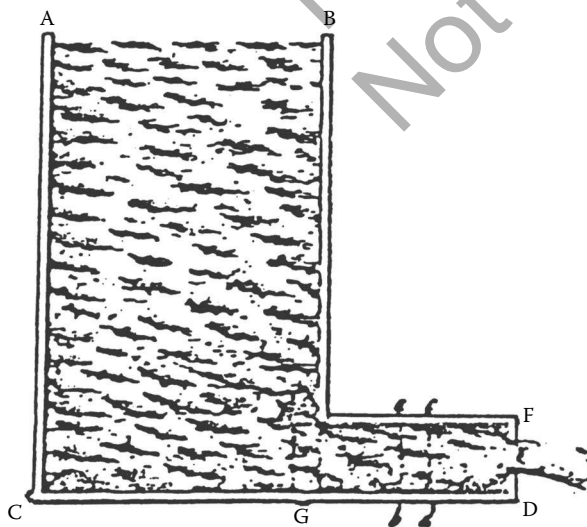


FIGURE 2.4 Conceptual water tank for demonstrating the Bernoulli principle. (Sketch by Daniel Bernoulli.)

“Bernoulli equation” we use today. However, Bernoulli went on to interpret the term VdV/dx as the pressure, which allows us to interpret the relation in *Hydrodynamica* as the form

$$p = \frac{a - V^2}{2c}$$

Since a and c are constants, this relation says qualitatively that, as velocity increases, pressure decreases.

From this, we are led to conclude the following:

1. The principle that pressure decreases as velocity increases is indeed presented in Bernoulli’s book, albeit in a slightly obscure form. Hence, it is clearly justified to call this the “Bernoulli principle,” as is done today. However, it is interesting to note that nowhere in his book does Bernoulli emphasize the importance of this principle, showing a certain lack of appreciation for its significance.
2. Bernoulli’s equation does *not* appear in his book, nor elsewhere in his work. *It is quite clear that Bernoulli never derived nor used Bernoulli’s equation.*

This is not to diminish Bernoulli’s contributions to fluid dynamics. His work was used as a starting point by other investigators in the eighteenth century. He was the first to examine the relation between pressure and velocity in a flow using the new scientific principles of the eighteenth century. As far as this author can ascertain, he was the first to use the elements of calculus to analyze a fluid flow, as illustrated in the differential equation shown above, obtained from his *Hydrodynamica*. His work inspired the work of other investigators, including that of Euler, d’Alembert, and Lagrange.

2.5 Henri Pitot and His Tube

A major advancement in experimental fluid dynamics occurred on November 12, 1732, at the Royal Academy of Sciences in Paris. On this day, Henri Pitot announced to the Academy a new invention by which he could directly measure the local flow velocity at a point in fluid. Later called the pitot tube, this device has become the most commonplace instrument in modern twentieth century fluid dynamic laboratories. Because of its importance, let us examine the historical details surrounding its development.

For Pitot himself, the invention of the Pitot tube was just one event in a reasonably productive life. Born in Aramon, France on May 3, 1695 of reasonably educated parents, Pitot’s youth was undistinguished; indeed, he demonstrated an intense dislike of academic studies. While serving a brief time in the military, Pitot was motivated by a geometry text published in a Grenoble bookstore, and subsequently spent 3 years at home studying mathematics and astronomy. In 1718, Pitot moved to Paris, and by 1723 had become an assistant in the chemistry laboratory of the Academy of Sciences. It was to this group that he delivered, on November 12, 1732, his announcement of his new device for measuring flow velocity—the Pitot tube.

His invention of the Pitot tube was motivated by his dissatisfaction with the existing technique of measuring the flow velocity of water, which was to observe the speed of a floating object on the surface of the water. So he devised an instrument consisting of two tubes; one was simply a straight tube open at one end that was inserted vertically into the water (to measure the static pressure) and the other was a tube with one end bent at right angles with the open end facing directly into the flow (to measure total pressure)—namely, the Pitot tube. In 1732, between two piers of a bridge over the Seine River in Paris, he used this instrument to measure the flow velocity of the river at different depths within the river. In his presentation to the Academy later that year, Pitot presented his results, which had importance beyond the Pitot tube itself. Contemporary theory, based on the experience of some Italian engineers, held that the flow velocity at a given depth in a river was proportional to the mass above it; hence the velocity was thought to increase with depth. Pitot reported the stunning (and correct) results, measured with his instrument, that in reality the flow velocity decreased as the depth increased. Hence, Pitot introduced his new invention with style. Later, in 1740, he accepted an invitation from the Estates General of Languedoc to supervise the draining of swamps in the province, which then led to his becoming director of public works of the province as well as superintendent of the Canal du Languedoc. In his old age, Pitot retired to his birthplace, and died at Aramon on December 27, 1771.

The development of the Pitot tube in 1732 was a substantial contribution to experimental fluid dynamics. However, in 1732, Henri Pitot did not have the benefit of Bernoulli's equation, which was obtained by Euler 20 years later. Pitot's reasoning for the operation of this tube was purely intuitive, and he was able to correlate by *empirical means* the flow velocity corresponding to the measured difference between the stagnation pressure as measured by his Pitot tube, and the flow static pressure, as measured by a straight tube inserted vertically in the fluid, with its open face tube parallel to the flow. As discussed in Anderson (1989), the proper application of Bernoulli's equation to extract the velocity from the Pitot measurement to stagnation pressure was not presented until 1913. In that year, John Airey at the University of Michigan published an exhaustive experimental behavior of Pitot tubes, and presented a rational theory for their operation based on Bernoulli's equation. Invented in the early part of the eighteenth century, the Pitot tube required two centuries before it was *properly* incorporated into fluid dynamics as a viable experimental tool.

2.6 The High Noon of Eighteenth-Century Hydrodynamics—Leonhard Euler and the Governing Equations of Fluid Motion

Today, in the modern world of twentieth century fluid dynamics, at the very instant that you are reading this page, there are literally thousands of fluid dynamicists who are solving the

governing equations of fluid motion for an inviscid flow. Such inviscid flows—flows without friction—adequately describe many aspects of practical fluid dynamic problems as long as friction is not being considered. These solutions may involve closed-form theoretical mathematics, or more likely today may involve direct numerical solutions on a high-speed digital computer. However, the governing equations that are being solved in such a “high-tech” fashion are themselves over 200 years old; they are called the *Euler equations*. The development of the Euler equations represents a contribution to fluid dynamics of a magnitude much greater than any other we have discussed so far in this chapter. They represent, for all practical purposes, the true beginning of theoretical fluid dynamics. These equations were first developed by Leonhard Euler; for this reason Euler is frequently credited as being the “founder of fluid mechanics.” This is somewhat of an overstatement, because as is almost always the case in physical science, Euler benefitted from earlier work, especially that of d'Alembert. On the other hand, Euler is a giant in the history of fluid dynamics, and his contributions bordered on the revolutionary rather than the evolutionary side. For these reasons, let us first take a look at Euler, the man.

Leonhard Euler was born on April 15, 1707 in Basel, Switzerland. His father was a Protestant minister who enjoyed mathematics as a pastime. Therefore, Euler grew up in a family atmosphere that encouraged intellectual activity. At the age of 13, Euler entered the University of Basel, which at that time had about 100 students and 19 professors. One of those professors was Johann Bernoulli, who tutored Euler in mathematics. Three years later, Euler received his master's degree in philosophy. It is interesting that three of the people most responsible for the early development of theoretical fluid dynamics—Johann and Daniel Bernoulli and Euler—lived in the same town of Basel, were associated with the same university, and were contemporaries. Indeed, Euler and the Bernoullis were close and respected friends—so much so that when Daniel Bernoulli moved to teach and study at the St. Petersburg Academy in 1725, he was able to convince the academy to hire Euler as well. At this invitation, Euler left Basel for Russia; he never returned to Switzerland, although he remained a Swiss citizen throughout his life.

Euler's interaction with Daniel Bernoulli in the development of fluid mechanics grew strong during these years at St. Petersburg. It was here that Euler conceived of pressure as a point property that can vary from point to point throughout a fluid, and obtain a differential equation relating pressure and velocity. In turn, Euler integrated the differential equation to obtain, for the first time in history, Bernoulli's equation in the form we use today. Hence, we see that Bernoulli's equation really is a misnomer; credit for it is legitimately shared by Euler.

When Daniel Bernoulli returned to Basel in 1733, Euler succeeded him at St. Petersburg as professor of physics. Euler was a dynamic and prolific man; by 1741 he had prepared 90 papers for publication and written the two-volume book *Mechanica*. The atmosphere surrounding St. Petersburg was conducive to such achievement. Euler wrote in 1749, “I and all others who had the good fortune to be for some time with the Russian Imperial

Academy cannot but acknowledge that we owe everything which we are and possess to the favorable conditions which we had there.”

However, in 1740, political unrest in St. Petersburg caused Euler to leave for the Berlin Society of Sciences, at that time just formed by Frederick the Great. Euler lived in Berlin for the next 25 years, where he transformed the society into a major academy. In Berlin, Euler continued his dynamic mode of working, preparing at least 380 papers for publication. Here, as a competitor to d'Alembert, Euler formulated the basis for mathematical physics.

In 1766, after a major disagreement with Frederick the Great over some financial aspects of the academy, Euler moved back to St. Petersburg. This second period of his life in Russia became one of physical suffering. In that same year, he became blind in one eye after a short illness. An operation in 1771 resulted in restoration of his sight, but only for a few days. He did not take proper precautions after the operation, and within a few days he was completely blind. However, with the help of others, he continued his work. His mind was as sharp as ever, and his spirit did not diminish. His literary output even increased—about half of his total papers were written after 1765!

On September 8, 1783, Euler conducted business as usual—giving a mathematics lesson, making calculations of the motion of balloons, and discussing with friends the planet Uranus, which had recently been discovered. At about 5:00 p.m., he suffered a brain hemorrhage. His only words before losing consciousness were “I am dying.” By 11 p.m., one of the greatest minds in history had ceased to exist.

Euler's contribution to theoretical aerodynamics were monumental; whereas Bernoulli and d'Alembert made contributions toward the physical understanding and the formulation of principles, Euler is responsible for the proper mathematical formulation of these principles, thus opening the door for future quantitative analyses of aerodynamic problems—analyses that continue on to the present day. The governing equations for an inviscid flow, incompressible or compressible, were presented by Euler in a set of three papers: *Principles of the Motion of Fluids* (1752), *General Principles of the State of Equilibrium of Fluids* (1753), and *General Principles of the Motion of Fluids* (1755). The successful derivation of these equations depended on two vital concepts that Euler borrowed in total or in part from previous researchers, as follows:

1. A fluid can be modeled as a continuous collection of infinitesimally small fluid elements moving with the flow, where each fluid element can change its shape and size continuously as it moves with the flow, but at the same time all the fluid elements taken as a whole constitute an overall picture of the flow as a continuum. The modeling of a flow by means of small fluid elements of finite size was suggested by Leonardo; however, the science and mathematics of Leonardo's time were not advanced enough for him to capitalize on this model. Later, Bernoulli suggested that a flow can be modeled as a series of thin slabs perpendicular to the flow; this is not unreasonable for the flow through a

duct such as the horizontal pipe at the bottom of Figure 2.4. However, the thin slab model lacks the degree of mobility that characterizes a small fluid element that can move along a streamline in three dimensions. A major advancement in flow modeling was made by D'Alembert; in 1744 he utilized a moving fluid element to which he applied the principle of mass conservation. Building on these ideas, Euler refined the fluid element model by considering an infinitesimally small fluid element to which he directly applied Newton's second law expressed in a form that utilized differential calculus. Indeed, this leads to the second point.

2. Newton's second law can be applied in the form of the following differential equation, which is a statement that force equals mass times acceleration, that is,

$$F = \frac{Md^2x}{dt^2}$$

In this differential equation, F is the force, M is the mass, and d^2x/dt^2 is the linear acceleration, that is, the second derivative of the linear distance, x . This is today the most familiar form of Newton's second law; it was first formulated in this form by Euler, and was documented in his paper titled *Discovery of a New Principle of Mechanics*, published in 1750.

Utilizing the two concepts listed above, namely that of an infinitesimally small fluid element moving along a streamline, and the application of both the principle of mass conservation and Newton's second law to the fluid element in the form of differential calculus as given above, Euler derived the partial differential equations of fluid motion that today carry his name, and that serve as the foundation for a large number of modern aerodynamic analyses. The equations derived by Euler in 1753 revolutionized the analyses of fluid dynamic problems. However, there was one important physical quantity missing from the Euler equations—friction. This leads to our next section.

2.7 Inclusion of Friction in Theoretical Fluid Dynamics: The Works of Navier and Stokes

At the beginning of the nineteenth century, the equations of fluid motion as derived by Euler were well known. However, these equations neglected an important physical phenomenon—a phenomenon that was appreciated by scientists in the eighteenth and nineteenth centuries but was not understood well enough to be properly included in any theoretical analysis—namely, friction. The governing flow equations that contain terms to account for friction are called the *Navier–Stokes equations*, named after the Frenchman Louis Marie Henri Navier (1785–1836) and the Englishman George Gabriel Stokes (1819–1903), who independently derived these equations in the nineteenth century. Almost 150 years later, the Navier–Stokes equations are still the fundamental equations used to analyze a viscous fluid flow. Moreover,

they are the subject of much research and application in the field of computational fluid dynamics today. Hence, the importance of the Navier–Stokes equations to modern fluid dynamics cannot be overstated.

The first accurate representation of the effects of friction in the general partial differential equations of fluid flow was given by Navier in 1822, as described in his papers titled *Memoire sur les lois du mouvement des fluides*, presented to the Paris Academy of Sciences. This was published 5 years later by the Academy. However, although Navier's equations were of the correct form, his theoretical reasoning was greatly flawed, and it is almost a fluke that he obtained the correct terms. Moreover, he did not fundamentally appreciate the true physical significance of what he had obtained. Before we explore these statements further, let us look at the man himself.

Claude Louis Marie Henri Navier was born in Dijon, France on February 10, 1785. His early childhood was spent in Paris, where his father was a lawyer to the Legislative Assembly during the French Revolution. After the death of his father in 1793, Navier was left under the care and tutelage of his mother's uncle, the well-known engineer Emiland Gauthey. (At the time of his death in 1806, Gauthey was considered France's leading civil engineer.) As a result of his granduncle's influence, Navier entered the Ecole Polytechnique in 1802, barely meeting the school's admission standards. However, within a year, Navier flowered, and he was among 10 students chosen to work in the field at Boulogne instead of spending his second year at the Polytechnique. In 1804, he entered the Ecole des Ponts et Chaussees, graduating in 1806 near the top of his class. During this time, he was influenced by the famous French mathematician, Jean Baptiste Fourier, whom Navier had as a professor of analysis. Fourier's impact on Navier was immediate and lasting. Within a short time, Navier became Fourier's protegee and lifetime friend.

During the next 13 years, Navier became a scholar of engineering science. He edited the works of his granduncle, who had died in 1806; these works represented the traditional empirical approach to numerous applications in civil engineering. In the process, Navier, based on his own research in theoretical mechanics, added a somewhat analytical flavor to the works of Gauthey. This, in combination with textbooks which Navier wrote independently for practicing engineers, introduced the basic principles of engineering science to a field which previously had almost been completely empirical. In fact, Navier is responsible for introducing the precisely defined concept of mechanical work in the analysis of machines. (Navier called the product of force times distance the "quantity of action.")

Because of his insistence on the importance and usefulness of engineering science in the solution of practical problems, Navier was given a teaching position at the Ecole des Ponts et Chaussees in 1819, where he permanently changed the style of teaching in engineering with his emphasis on physics and analyses. In 1831, he replaced the famous mathematician, Augustin Louis de Cauchy at the Ecole Polytechnique. For the rest of his life, Navier lectured at the university, wrote books, and at times practiced his profession of civil engineering, particularly the design of bridges.

(It is ironic that the bridge design that brought him the most public notice collapsed before it was totally constructed. This was a suspension bridge over the river Seine in Paris. Toward the end of construction on the bridge, a sewer near one pier ruptured, flooding the area, weakening the foundation of the pier, and causing the bridge to sag. The damage could have been easily repaired. However, for various political and economic reasons, the Municipal Council of Paris had been opposed to building Navier's bridge. The listing of the bridge due to the sewer failure gave the Council the opportunity to lobby for halting the project. The Council was successful, the bridge was torn down, and Navier was greatly disappointed. Here is one of many examples in history where engineering competence is no match for fate and politics—even for a person as well respected as Navier.)

Bridges notwithstanding, history will recognize Navier as the first to derive the governing equations for fluid flow including the effects of friction. However, there is irony here too. Navier had no concept of shear stress in a flow (i.e., the frictional shear stresses acting on the surface of a fluid element). Rather, he was attempting to take Euler's equations of motion and modify them to take into account the forces that act between molecules in the fluid. He assumed these intermolecular forces to be repulsive at close distance, and attractive at larger distances away from the molecule; thus, for a fluid that is stationary, the spacing between molecules is a result of the equilibrium between the repulsive and attractive forces. Carrying through an elaborate derivation using this model, Navier produced a system of equations that were identical to Euler's equations of motion, except for additional terms that appeared due to the intermolecular forces. For the mathematically versed readers, these terms as derived by Navier involved second derivatives of velocity multiplied by a constant, where the constant simply represented a function of spacing between the molecules. This is indeed the proper form of the terms involving frictional shear stress, namely a second derivative of velocity multiplied by a coefficient called the viscosity coefficient. The irony is that, although Navier had no concept of shear stress and did not set out to obtain the equations of motion including friction, he nevertheless obtained the proper form of the equations for flow with friction. Later in the nineteenth century, this form was indeed recognized as proper for a frictional flow, and that is why the governing equations for flow with friction today are called, in part, the Navier–Stokes equations. However, Navier did not appreciate the true significance of his result; indeed, he did not attribute any physical significance whatsoever to the constant multiplying the second derivatives of velocity—the constant that later was clearly identified as the coefficient of viscosity. (This author notes parenthetically that, in the final analysis, Navier's results were not totally a fluke. Our modern understanding of the physical significance of the viscosity coefficient comes from a study of the kinetic theory of gases, from which we can easily show that the viscosity coefficient is directly proportional to the molecular mean free path—the mean distance a molecule moves in between successive collisions with other molecules. Hence, Navier's approach wherein he was accounting for the spacing between molecules due to the

balance between attractive and repulsive intermolecular forces is not totally off the mark, although the mean free path and the mean spacing between molecules are different values—right church but wrong pew.)

Although Navier did not appreciate the real physical significance of his equations for a fluid flow, one of his contemporaries did: Jean Claude Barre de Saint-Venant. Born in Villiers-en-Biere, Leine de-Marne, France on August 23, 1797, Saint-Venant was educated at the Ecole Polytechnique, graduating in 1816, 12 years after Navier finished at the same school. Saint-Venant then joined the Service des Poudres et Salpêtres, and in 1823 moved to the Service des Ponts et Chaussées. Here he served for 20 more years, after which he retired to a life of teaching and research. He died at the age of 92, after a long and productive life, on January 6, 1886, at St. Ouen, Loir-et-Cher, France. Saint-Venant was one generation younger than Navier, both in age and professional stature. Navier was elected to the Paris Academy of Sciences in 1824; Saint-Venant became a member in 1868. However, Saint-Venant was quite familiar with Navier's work, as reflected in his book *Mecanique Appliquee de Navier, Annotee par Saint-Venant*, published in Paris in 1858. Seven years after Navier's death, Saint-Venant published a paper at the Academy of Sciences wherein he rederived Navier's equations for a viscous flow considering internal viscous stresses—eschewing completely Navier's molecular model approach. Appearing in the year 1843, this paper was the first to properly identify the coefficient of viscosity and its role as a multiplying factor with velocity gradients in the flow. He further identified these products as viscous stress acting within the fluid due to the influence of friction. Hence, in 1843, Saint-Venant had got it right, and had recorded it. Why it is that his name is never associated with these equations is a mystery to this author, and simply has to be accepted as a miscarriage of technical proprietorship.

This leads up to Sir George Gabriel Stokes, who was just a few hundred miles away from Navier and Saint-Venant, across the English Channel, but who was light years away in terms of familiarity with the work of these Frenchmen. George Stokes is the second-half namesake of the Navier–Stokes equations. Before we examine why, let us first look at the man himself.

Stokes was born in Skreen, Ireland, on August 13, 1819. The hallmark of his family was religious vocations; his father was the rector of the Skreen parish, his mother was the daughter of a rector, and ultimately all three of his brothers became ministers in the church. Throughout his life, George Stokes remained a strongly religious person. Indeed, toward the end of his life, he became interested in the relationship of science to religion; from 1886 to the year of his death in 1903, he was president of the Victoria Institute of London, a society for examining the relationship between Christianity and contemporary thought, with an emphasis on science. During his childhood, Stoke's education began with tutoring from his father, which led to his admission to Bristol College in Bristol, England. At Bristol, he prepared for university studies, and entered Pembroke College, Cambridge, at the age of 18. Stokes was a highly intelligent man; at the time

of graduation from Cambridge, he was immediately elected to a fellowship in Pembroke College. Eight years later, Stokes occupied the Lucasian Chair at Cambridge, the same professorship held by Newton almost two centuries earlier. Since the Lucasian endowment was small, Stokes had to simultaneously take a second position in the 1850s, teaching at the Government School of Mines in London. He held the Lucasian Chair until his death at Cambridge on February 1, 1903.

Fluid dynamicists think of George Stokes and they visualize a man who made a momentous, fundamental contribution to the discipline via his derivation and subsequent use of the equations which today are called the Navier–Stokes equations. These equations are the most fundamental descriptors of a general three-dimensional, unsteady, viscous fluid flow; they are the foundation of modern theoretical and computational fluid dynamics. However, if Stokes were alive today, he would most likely feel more comfortable in being identified as a physicist and to some small degree a mathematician who had made substantial contributions in the area of optics. Beginning about 1845, he worked on the propagation of light and how it interacted with the ether—a continuous substance surrounding the earth according to the prevailing theory of that day. It is interesting to note that Stokes analyzed the properties of the hypothetical ether using an analogy with his fluid dynamic equations of motion. He concluded that if the earth moved through a stationary ether, the ether must be a very rarefied fluid. In a contradictory sense, he also concluded that the propagation of light required the ether to be much like a very elastic solid. Hence, one of the first theoretical consequences of the Navier–Stokes equations was not a definitive flowfield calculation (as used today), but rather an inconclusive study of the properties of the ether. To make things more inconclusive, Stokes showed in 1846 that the laws of reflection and refraction remained unchanged whether or not an ether existed. Of much greater importance in the physics of light was Stokes' work on fluorescence, the phenomenon wherein a substance absorbs electromagnetic waves on one wavelength, and emits waves of another wavelength. In particular, he made observations of the blue light emitted from the surface of an otherwise transparent and colorless solution of sulfate of quinine when the solution is irradiated by invisible ultraviolet rays. His physical explanation of this process won him the Rumford Medal of the Royal Society in 1852; indeed, he coined the word “fluorescence” in the context of his explanation. Later, he suggested the use of fluorescence to study the properties of molecules, and is credited as the first to develop the principles of spectrum analysis. In summary, the point made here is that Sir George Stokes would most likely credit himself for contributions in optics rather than fluid dynamics. In this sense, there is some irony in the fact that today his name is literally invoked by fluid dynamicists much more frequently than by those working in any other field of science and engineering.

With this as background, we now focus on Stoke's contributions in fluid dynamics. He was unfamiliar with the work of Navier and Saint-Venant in France, and was not aware of their

derivations of the equations of motion for a fluid with friction. Quite independently, he utilized the concept of internal shear stresses in a moving fluid, and derived the governing equations of a viscous fluid (a fluid with internal friction). His derivation of the equations was much like the way they are derived today; in the process, he properly identified the dynamic viscosity coefficient, μ , as it appears in the Navier–Stokes equations. This work was published in 1845 (2 years after Saint-Venant’s similar derivation) in his paper titled *On the Theories of the Internal Friction of Fluids in Motion, and of the Equilibrium and Motion of Elastic Solids*. As with most scientists studying fluid dynamics in the nineteenth century, Stokes dealt with an incompressible flow. For such flows, the energy equation is not essential. With this one exception, the work of Stokes remains unchanged to the present day. The fundamental equations for a flow with friction—the Navier–Stokes equations—were therefore well established more than 150 years ago. This should be a sobering thought for modern fluid dynamicists, and especially for those at the cutting edge of modern computational fluid dynamics, who deal with the Navier–Stokes equations on an almost daily basis. Here, we are using ultramodern supercomputers to solve equations that are covered by the dust of ages, but that have nonetheless weathered the test of time.

2.8 Final Comment

With this, we bring to a close our brief look at a few aspects of the history of fluid dynamics. We have examined only a few peaks in the whole mountain range of the subject. If you are interested in learning more about the historical development of fluid dynamics, the books listed in the references below are recommended.

References

- Anderson, J. D., Jr., *Introduction to Flight*, 3rd Edition, McGraw-Hill, New York, 1989.
- Anderson, J. D. R., *A History of Aerodynamics and Its Impact on Flying Machines*, Cambridge University Press, New York, 1997.
- Giacomelli, R., The aerodynamics of Leonardo da Vinci, *J. Royal Aeronaut. Soc.*, 34(240), 1016–1038, 1920.
- Giacomelli, R. and Pistolesi, E., Historical sketch, in *Aerodynamic Theory*, Vol. I, W. F. Durand, Ed., Julius Springer, Berlin, 1934, 305–394.
- Rouse, H. and Ince, S., *History of Hydraulics*, Iowa State Institute of Hydraulic Research, Iowa City, Iowa, 1957.
- Tokaty, G. A., *A History and Philosophy of Fluid Mechanics*, G. T. Foulis and Co., Henley-on-Thames, England, 1971.

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