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Integrated Production Planning Model for Noncyclic Maintenance and Production Planning

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1.1 Introduction

Maintenance and production are closely related in different ways. This relationship makes production planning and maintenance planning the most important and demanding areas in the process of decision-making by industrial managers. Hence, they have been the focus of attention in the manufacturing industry while a lot of research has also been devoted to them in the area of operations research. Although the two activities are interdependent, they have most often been performed independently. Integration of production planning and maintenance planning into one single problem is a complex and challenging task since the resulting integrated planning problem leads to nonoptimal solutions.

Maintenance becomes necessary because of either a failure in production or the undesirably low quality of the items produced. However, the significance of maintenance planning can be more vividly realized when maximum plant availability and maximum mean time between equipment failures are sought at the lowest cost.

Maintenance activities may be classified into four types: corrective maintenance, predictive maintenance, repairs maintenance, and preventive maintenance (PM). Corrective maintenance can be defined as the maintenance that is required when an item has failed or worn out, to bring it back to working order. While predictive maintenance tends to include direct measurement of the item, repairs maintenance is simply doing maintenance work as need develops. This elementry approach has sometimes been replaced by periodic overhauls and other preventive maintenance activities. PM is performed periodically in order to reduce the incidence of equipment failure and the costs associated with it. These costs include disrupted production schedules, idled workers, loss of output, and damage to products or other equipment. PM, thus, improves production capacity, production quality, and overall efficiency of production plants. Moreover, it can be scheduled to avoid interference with production.

There are trade-offs between PM planning and production planning. PM activities take time that could otherwise be used for production, but delaying PM for production may increase the probability of machine failure. Whenever an unexpected machine failure occurs, the current production plan becomes inadequate and needs to be modified. Changes in production plan sometimes cause extra costs or significant changes in the service level and production line productivity.

Production planning mainly has two aspects: lot-sizing and scheduling. Lot-sizing concerns determining production quantity while scheduling concerns sequencing products on the production line. Decisions for these two problems are mostly made in a hierarchical manner. In other words, the lot-sizing problem is solved first and the output is used in the sequencing and scheduling problem. The problem is sometimes described as the general lot-sizing and scheduling problem; GLSP (Fleischmann and Meyr 1997) or capacitated lot-sizing problem with sequence-dependent setup times (CLSP-SD) that addresses the integrated lot-sizing and scheduling problems simultaneously due to their dependencies. In this chapter, a new integrated model is presented for the noncyclic maintenance and production planning problem. The Markov chain is used for producing the parameters required for processing the model of a single-stage multiparallel machine production system with the objective of maximizing profits with the assumption of demand flexibility. In this model, the value of maintenance has been taken into account. The product yield depends on equipment conditions, which deteriorate over time. The objective is to determine equipment maintenance schedule, demand quantity, and lot-sizes, and production schedules in such a way that the expected profit is maximized.

1.2 Literature Review

PM planning models are typically stochastic models accompanied by optimization techniques designed to maximize equipment availability or minimize equipment maintenance costs. There are mathematical or simulation models.

The literature abounds in papers on planning and optimizing maintenance activities. However, a few can be found dealing with models that combine PM planning and production planning.

The models reported in the literature include such decision variables as the number of maintenance activities, safety buffers, and inspection intervals. While the objective in most models is minimizing costs, some also consider system lifetime, which is generally assumed to have a Weibull distribution.

Models developed for integrating PM planning and production planning are Np-Hard, which can be optimally solved for small-size instances, but obtaining optimal solutions is impractical for large-size instances. This, therefore, warrants efficient solvers for large-size problems. Both heuristic and meta-heuristic methods including genetic algorithm, simulated annealing algorithm, and Lagrangian procedure, and expert systems have been used for the solution of these models.

Most papers in this area deal with production scheduling, and the models used for production scheduling and PM planning are designed with an implicit common goal of maximizing equipment productivity. Some studies extend the simple machine scheduling models by considering the maintenance decisions as given, or as constraints, rather than integrating them. The problem in these studies is modeled as a sequencing and scheduling problem with the machine availability constraint (Molaee et al. 2011).

Different methods have been used to develop models for the production planning and PM problems. Cassady and Kutanoglu (2005) have classified these methods into two broad approaches: reactive and robust. In the reactive approach, attempts are made to update production when a failure occurs. In robust planning, the plan is not sensitive to failure events. In another classification (Iravani and Duenyas 2005), the studies are classified into two groups. While the first focuses on the effect of failure on production schedule, the other group integrates production and maintenance planning into a single problem. Meller and Kim (1996) reviewed the literature and classified studies into two categories: one focusing on PM, and the other focusing on the statistical analysis of safety-stock-based failure neglecting PM.

Brandolese et al. (1996) considered a single-stage, multiproduct production environment with flexible parallel machines. They developed an expert system for the planning and management of a multiproduct and one-stage production system made up of flexible machines operating in parallel. The system schedules both production and maintenance at the same time.

Setup costs are sequence dependent. Sloan and Shanthikumar (2000) studied a multiproduct, single-machine problem in which the machine has states that change during the planning horizon such that the machine state affects the production rate of each product. They used the Markov chain and their objective function aimed to maximize profits. In each period, either a product is being produced or a maintenance activity is being performed. Their model determines the optimal policy of production and maintenance. The objective is achieving optimal maintenance policy in such a way that the sum of the discounted costs of maintenance, repairs, production, backorders, and inventory is minimized.

Aghezzaf and Najid (2008) presented a production plan and a maintenance plan in a multiproduct, parallel machine system with corrective maintenance and PM. It is assumed that when a production line fails, a minimal repair is carried out to restore it to an *as-bad-as-old* status. PM is also carried out periodically at the discretion of the decision maker to restore the production line to an *as-good-as-new* status. The resulting integrated production and maintenance planning problem is modeled as a nonlinear mixed-integer program when each production line implements a cyclic PM policy. When noncyclic PM policies are allowed, the problem is modeled as a linear mixed-integer program. In this situation, maintenance activities decrease production capacity. Sitompul and Aghezzaf (2011) proposed an integrated production and maintenance hierarchical plan. Noncyclic maintenance in the single machine problem has been considered in another study (Nourelfath et al. 2010), in which it is assumed that while production capacity is constant, a decision must be made in each period about implementing the PM.

Fitouhi and Nourelfath (2012) extended upon previous studies (Nourelfath et al. 2010). They proposed a model in which the noncyclic maintenance assumption was abandoned and the assumption that the machine has several states due to its components was adopted instead. The proposed model coordinates the production with the maintenance decisions so that the total expected cost is minimized. We are given a set of products that must be produced in lots on a multistate production system during a specified finite planning horizon. Planned PM and unplanned corrective maintenance can be performed on each component of the multistate system. The maintenance policy suggests cyclic preventive replacements of components and a minimal repair on failing components. The objective is to determine an integrated lot-sizing and PM strategy of the system that will minimize the sum of preventive and corrective maintenance costs, setup costs, holding costs, backorder costs, and production costs, while satisfying the demand for all products over the entire horizon. Production yield is influenced by the machine state.

Yao et al. (2005) studied the joint PM and production policies for an unreliable production-inventory system in which maintenance/repair times are nonnegligible and stochastic. A joint policy decides (1) whether or not to perform PM and (2) if PM is not performed, then how much to produce. A discrete-time system is considered, and the problem is formulated as a Markov decision process model. Although their analysis indicates that the structure of the optimal joint policies is generally very complex, they were able to characterize several properties regarding PM and production including optimal production/maintenance actions under backlogging and high inventory levels. Wee and Widyadana (2011) studied the economic production quantity models for deteriorating items with rework and stochastic PM time.

Lu et al. (2013) studied system reliability. According to them, a system reliability lower bound is determined that is smaller than the system reliability. Marais and Saleh (2009) investigated the maintenance value and proposed that maintenance has an intrinsic value. They argue that the existing cost-oriented models ignore an important dimension of maintenance activities that involves quantifying their value. They consider systems that deteriorate stochastically and exhibit multistate failures. The state evolution is modeled in their study using the Markov chain and directed graphs. To account for maintenance value, they calculate the net present value of maintenance in their model.

Njike et al. (2011) used the value-optimized concept in their research. They sought to develop an optimal stochastic control model in which interactive feedback consisted of the quantity of flawless and defective products. The main objective was to minimize the expected discounted overall cost due to maintenance activities, inventory holding, and backlogs. A near-optimal control policy of the system was then obtained through numerical techniques. The originality of their research lies in the fact that all operational failures have been taken into account in the same optimization model. This brings a value added to the high level of maintenance and for operation managers who need to consider all failure parameters before taking cost-related decisions.

1.3 Integrated Model for Noncyclic Maintenance Planning and Production Planning

1.3.1 Problem Statement

In this section, an integrated model is presented for the noncyclic maintenance planning and production planning problem. The objective is to maximize profits. The model considers simultaneous lot-sizing and scheduling. The challenge commonly faced within production planning is the coordination of demand and production capacity.

Demand flexibility assumption is also introduced into the model. In many industries, product yield is heavily influenced by equipment conditions. Previous studies have focused either on maintenance at the expense of the effect of equipment conditions on yield or on production at the expense of the possibility for actively changing machine state.

1.3.2 Assumptions

The assumptions made here are classified into those related to production planning and those concerning PM planning.

Assumptions of production planning:

- The model is a multiproduct one.
- The model is in a multiparallel machine environment.
- The available capacity is finite. Considering PM planning in each period, the capacity may take different values as computed based on mathematical expectation.
- The planning horizon is finite and consists of T periods.
- The demand for a product is not known before each period and is determined by the model. For each product, this value ranges between a lower and an upper bound for the demand.
- Shortage is allowed in periods. However, the total demand should be met at the end of the planning horizon.
- Setup times and setup costs are sequence dependent.
- Holding costs, setup costs, and production costs are time independent.
- The model has the characteristic of setup preservation. It means that if we have an idle time, the setup state would not change after it.
- Lots are continuous. This means that production can continue for the next period with no break and with no setup.
- The setup state is specific at the beginning of the planning horizon.
- It is possible to produce some types of products in each period. In other words, the model is a big-time bucket one.
- The objective function is maximizing the sales revenue minus production, holding, and setup costs.
- The breakdown of setup time between two periods is not allowed, and the setup is finished in the same period in which it begins.

Assumptions of PM planning:

- The machine has the following three states:
  1. Working at good efficiency (state 1).
  2. Working at low efficiency (state 2).
  3. Where the machine breaks down, a non-PM repair state starts after a sudden breakdown (state r).
- Product quality is not influenced by the machine state.
- Maintenance operation does not create a disturbance or a change in the setup state.
- The PM operation is an activity with a positive effect; it increases system efficiency. There is at least one state in which the PM improves system efficiency.
- For PM planning, microperiods are considered to be separate from microperiods in production planning. Therefore, the PM schedule is discrete.
- In each microperiod, it is decided whether one and only one PM is to be performed or not.
- The efficiency or the capacity of a machine is reduced by production or as a result of exploiting this capacity. Therefore, the state of the machine goes toward state 2 or state *r*.
- The state of the machine turns to state 1 after a PM operation.
- Both the PM operation and the emergency maintenance operation are costly. In addition, they reduce the capacity of the period as they use this capacity.
- Only one PM operation is possible in each maintenance microperiod. On the other hand, it is assumed that only one sudden breakdown may happen in each microperiod.
- Transition of machine state is memory less from one period to the next.

In addition to these assumptions, the proposed model considers the existence of triangular inequality conditions or their paucity. In most industries, setup times conform to the triangular inequality. This assumption, which can also be applied to costs, is stated as follows:

$$Sc_{ik} + Sc_{kj} > Sc_{ij} \tag{1.1}$$

where Sc_{ij} represents the setup time from product *i* to product *j*.

In simple words, the triangular inequality states that the setup time or the setup cost for moving directly from product *i* to product *j* is less than the time or cost when there is a mediator.

In some industries, it is plausible that setup times do not conform to the triangular inequality. Changing color in some industries can be mentioned as an example; changing color from black to white needs more setup time than changing color from black to blue then from blue to white.

1.3.3 Profit Maximization of General Lot-Sizing and Scheduling Problem

The proposed integrated model for maintenance planning and production planning is based on the profit maximization general lot-sizing and scheduling problem (PGLSP) model (Sereshti 2010). Recently, attention has been directed toward the simultaneous lot-sizing and scheduling problem that has come to be called the general lot-sizing and scheduling problem or GLSP. For modeling this problem, two distinctive approaches may be employed. In the first approach, there are two kinds of time buckets, small buckets and large ones. Small buckets or positions are within the large buckets or macroperiods. The positions, or microperiods, are used for sequencing. This approach was first presented by Fleischmann and Meyr (1997). Meyr (2000) extended GLSP to deal with sequence-dependent setup times. The second approach is based on the CLSP-SD, which is related to the traveling salesman problem (Almada-Lobo et al. 2008).

The profit maximization of GLSP with demand choice flexibility is an extension of the GLSP in which the assumption of flexibility in choosing demands is also included. The accepted demand in each period can vary between its upper and lower bounds. The upper bound could be the forecasted demand, and the lower bound is the organization commitments toward customers or minimum production level according to production policy. PGLSP can be described as follows.

Having P products and T planning periods, the decision maker seeks to determine (1) the accepted demand of each product in each period, which is between an upper and a lower bound, (2) the quantity of lots for each product, and (3) the sequence of lots. The objective

Table 1.1 Parameters

T	Number of planning periods	
P	Number of products	
N	Number of positions in planning horizon	
π_n	The period in which position n is located	$n = 1,\ldots,N$
C_t	Available capacity in each period	$t = 1,\ldots,T$
Ld_{jt}	Demand lower bound for product j in period t	$j = 1,\ldots,P, t = 1,\ldots,T$
Ud_{jt}	Demand upper bound for product j in period t	$j = 1,\ldots,P, t = 1,\ldots,T$
n_t	Number of positions in period t	$t = 1,\ldots,T$
F_t	First position in period t	$t = 1,\ldots,T$
L_t	Last position in period t	$t = 1,\ldots,T$
h_j	Holding cost for one unit of product j	$j = 1,\ldots,P$
r_{jt}	Sales revenue for one unit of product j in period t	$j = 1,\ldots,P, t = 1,\ldots,T$
Cp_j	Production cost for one unit of product j	$j = 1,\ldots,P$
p_j	Processing time for one unit of product j	$j = 1,\ldots,P$
S_{ij}	Setup cost for transition from product i to product j	$i = j = 1,\ldots,P$
St_{ij}	Setup time for transition from product i to product j	$i = j = 1,\ldots,P$
I_{j0}	Initial inventory level for product j	$j = 1,\ldots,P$

function is maximizing the sales revenues minus production, holding, and setup costs. Backlog is not allowed. Setup times and costs are sequence dependent. The triangular inequality lies between setup times. Back order is not allowed.

The parameters for this model are presented in Table 1.1.

This model is an extension of the model proposed by Meyr (2000). PGLSP has also been modeled through the traveling salesman problem approach (Sereshti and Bijari 2013). In Meyr's model, the microperiods or positions within the planning periods are used as a modeling consideration to define the sequence of products. The number of these microperiods in each macroperiod forms a parameter of the model, and they are used to define the first and last positions in each period. The decision variables for this model are as follows:

I_{ij} = Inventory level of product j at the end of period t.

D_{jt} = Accepted demand of product j in period t.

Q_{jt} = Quantity of product j produced in position n.

Y_{jt} = A binary variable that is 1 when the setup state in position n is for product j.

X_{ijn} = A positive variable whose amount is always 0 or 1. This variable is 1 when the setup state changes from product i to product j in position n.

The mathematical model is presented as follows:

$$\text{Max} \sum_{j=1}^{P}\sum_{t=1}^{T} r_{jt} D_{jt} - \sum_{j=1}^{P}\sum_{n=1}^{N} Cp_j Q_{jn} - \sum_{j=1}^{P}\sum_{i=1}^{P}\sum_{n=1}^{N} S_{ij} X_{ijn} - \sum_{j=1}^{P}\sum_{t=1}^{T} h_j I_{jt}$$

subject to

$$I_{jt} = I_{j(t-1)} + \sum_{n=F_t}^{L_t} Q_{jn} - D_{jt} \quad j = 1,\ldots,P, \quad t = 1,\ldots,T \tag{1.2}$$

$$Ld_{jt} \le D_{jt} \le Ud_{jt} \quad j = 1,\ldots,P, \quad t = 1,\ldots,T \tag{1.3}$$

$$Q_{jn} \le M_{j\pi_n} Y_{jn} \quad j = 1,\ldots,P, \quad n = 1,\ldots,N \tag{1.4}$$

Other constraints of the model are the same as the Meyr's model.

The objective function of model is to maximize sales revenues minus production, setup, and holding costs. Constraint (1.2) shows the balance among demand, production, and inventory. Constraint (1.3) guarantees that the accepted demand for each product in each period is between its upper and lower bounds. Constraint (1.4) ensures that a product can be produced when its setup is complete. The upper bound of production in this constraint can be seen in statement (1.5). If we just use c_t/p_j as the upper bound, the constraint will be true; using the maximum value of the remaining demand in the following period may result in a tighter constraint, which occurs when the remaining demand is less than the production capacity.

$$M_{jt} = \min\left\{\frac{C_t}{p_j}, \sum_{k=t}^{T} Ud_{jk}\right\} \quad j = 1,\ldots,P, \quad n = 1,\ldots,N \tag{1.5}$$

1.3.4 Integrated Model

We have used PGLSP for modeling our problem (Bijari and Jafarian 2013). In the model, both macroperiods and microperiods are considered as in the basic GLSP. The decision maker wants to define the mentioned decisions in the previous section and also the period in which PM will be executed. The objective function is maximizing sales revenues minus production, holding, shortage costs, setup, PM, and non-PM costs.

The model is stochastic because the machine state is stochastic, too. The objective is maximizing the expected value of profits. Machine has three states. We use production microperiods for producing products. Maintenance microperiod is used for performing maintenance activities. In each period, the probability of machine state after the last PM can be determined. The parameters of the production microperiods are as follows:

- R: Number of products
- π_n: Number of microperiods containing position n
- n_t: Number of positions available in period t
- nr_t: Number of maintenance microperiods in period t
- β: Coefficient of efficiency when the machine is in state 2
- Cp_j: Production cost of j
- ρ_{jk}: Usage rate of machine k for producing item j
- S_{ijk}: Setup cost of machine k for producing item j after item i
- St_{ijk}: Setup time of machine k for producing item j after item i
- e: Discount rate in each period
- b_j: Shortage cost in each period
- cr: Non-PM cost
- m_j: Minimum production lot-size of j
- $P_i^{\tau r}$: Probability of state 1 after $(\tau r - 1)$ microperiods from last PM
- π_{nr} : Number of macroperiods that include maintenance microperiod nr
- Crp: PM cost
- U_k: Maintenance microperiod capacity for machine k
- δ_{ij}: Probability of transition from state i to j

The model and its decision variables are proposed as follows:

- I_{jt}^{+}: Inventory of product j at the end of period t
- I_{jt}^{-}: Shortage of product j at the end of period t
- Q_{jnk}: Production quantity of product j in microperiod n for machine k

tr_{nrk}: The number of microperiods (plus 1) between the last PM and the maintenance period nr for machine k

Y_{jnk}: Binary variable; it is 1 if product j is produced in position n on machine k

X_{ijnk}: If machine K setup is accomplished for producing product j in position n; when product i is produced in this position, it is 1; otherwise, zero $j, i = 1, \ldots, R$, $k = 1, \ldots, K$, $n = 1, \ldots, N$

q_k^{nr}: Binary variable; PM was done (1) or was not done in the maintenance microperiod nr on machine k

Other parameters and decision variables are the same as PGLSP. Production microperiods parameters are expressed as follows:

F_{tk}: The first position in period t for machine k, $k = 1, \ldots, K$, $t = 1, \ldots, T$

L_{tk}: The last position in period t for machine k

N: Total number of positions at planning horizon

$$F_t = \sum_{k=1}^{t-1} n_k + 1 \tag{1.6}$$

$$L_t = F_t + n_t - 1 \tag{1.7}$$

$$N = \sum_{t=1}^{T} n_t \tag{1.8}$$

Maintenance (M) microperiods parameters are as follows:

F_{rtk}: The first PM position in period t for machine k

L_{rtk}: The first PM position in period t for machine k

Nr: Total number of PM positions

$$F_{rt} = \sum_{k=1}^{t-1} nr_k + 1 \tag{1.9}$$

$$L_{rt} = F_{rt} + nr_t - 1 \tag{1.10}$$

$$Nr = \sum_{t=1}^{T} nr_t \tag{1.11}$$

The model is shown as follows:

$$\max E(Z) = \sum_{j=1}^{R}\sum_{t=1}^{T}(1-e)^{t-1} r_{jt} D_{jt} - \sum_{j=1}^{R}\sum_{n=1}^{N}\sum_{k=1}^{K}(1-e)^{t-1} cp_j Q_{jnk}$$

$$-\sum_{j=1}^{R}\sum_{i=1}^{R}\sum_{n=1}^{N}\sum_{k=1}^{K}(1-e)^{\pi_n -1} S_{ij} X_{jink} - \sum_{j=1}^{R}\sum_{t=1}^{T}(1-e)^{t-1} h_j I_{jt}^{+}$$

$$-\sum_{j=1}^{R}\sum_{t=1}^{T}(1-e)^{t-1} b_j I_{jt}^{-} - cr\sum_{k=1}^{K}\sum_{nr=1}^{Nr}\sum_{\tau r}^{Nr}\frac{(1-e)^{\pi_{nr}-1} Pr^{\tau r}}{\left(tr_{nrk} - tr\right)M + 1}$$

$$-crp\sum_{k=1}^{K}\sum_{nr=1}^{Nr}(1-e)^{\pi_{nr}-1} q_k^{nr} \tag{1.12}$$

subject to

$$I_{jt}^{+} = I_{j(t-1)}^{+} - I_{j(t-1)}^{-} + \sum_{k=1}^{K}\sum_{n=F_{tk}}^{L_{tk}} Q_{jnk} - D_{jt} + I_{jt}^{-} \quad \forall t, j \tag{1.13}$$

$$\sum_{k=1}^{K}\sum_{n=1}^{N} Q_{jnk} = \sum_{t=1}^{T} D_{jt} \quad \forall j \tag{1.14}$$

$$Ld_{jt} \le D_{jt} \le Ud_{jt} \quad \forall t, j \tag{1.15}$$

$$Q_{jnk} \le MY_{jnk} \quad \forall j, n, k \tag{1.16}$$

$$\sum_{j=1}^{R}\sum_{n=F_{tk}}^{L_{tk}} \rho_{jk} Q_{jnk} + \sum_{i=1}^{R}\sum_{j=1}^{R}\sum_{n=F_{tk}}^{L_{tk}} St_{ijk} X_{ijnk}$$

$$\le \sum_{nr=F_{rtk}}^{L_{rtk}}\sum_{\tau r=1}^{Nr}\frac{\left[P_1^{\tau r} U_k + P_2^{\tau r} U_k^{\beta} \dagger\right]}{\left(tr_{nrk} - \tau r\right)M + 1} \quad \forall k, t \tag{1.17}$$

$$\sum_{j=1}^{R} Y_{jnk} = 1 \quad \forall n, k \tag{1.18}$$

$$X_{ijnk} \ge Y_{i(n-1)k} + Y_{jnk} - 1 \quad \forall j, i, n, k \tag{1.19}$$

Table 1.2 Transition Matrix

	1	2	R
1	δ_{11}	δ_{12}	δ_{1r}
2	0	δ_{22}	δ_{2r}
3	1	0	0

$$tr_{nrk} = tr_{(nr-1)k}\left(1-q_k^{nr}\right)+1 \quad \forall n,k \tag{1.20}$$

$$Y_{jnk}, \quad q_k^{nr} \in \{0,1\} \quad \forall j,n,k,nr \tag{1.21}$$

$$X_{ijnk}, Q_{jnk}, I_{jt}^{+}, I_{jt}^{-}, tr_{nrk}, D_{jt} \geq 0 \quad \forall j,n,k,i \tag{1.22}$$

The transition matrix is shown in Table 1.2.

The probability of state i after ($\tau r - 1$) microperiods from last PM can be obtained by the following equations:

$$P_1^{tr} = P_1^{tr-1}\delta_{11} + P_r^{(tr-1)}$$

$$P_2^{tr} = P_2^{tr-1}\delta_{22} + P_1^{tr-1}\delta_{12}$$

$$P_r^{tr} = P_2^{tr-1}\delta_{2r} + P_1^{tr-1}\delta_{1r}$$

$$P_1^1 = P_2^1 = P_r^1 = 0$$

$$P_1^2 = 1, \quad P_2^2 = P_r^2 = 0$$

All P_i^1 $(tr = 1)$ are equal to zero when PM is performed on the machine.

A discount rate is used in the objective function. It designates the value of maintenance. The first term in the objective function is related to sales revenue. Other terms designate production cost, setup cost, holding cost, shortage cost, expected value of non-PM cost, and PM cost. Non-PM cost is determined by multiplying the emergency non-PM cost by the probability of this state ($P_r^{\tau r}$) after $\tau r - 1$ from the last PM period. The denominator ensures that the value of τr is properly chosen. Only when tr equals tr_{nrk}, the denominator equals 1; otherwise, it has a big value because M is a big value. Thus, fractions become zero.

Constraint (1.13) shows production, demand, inventory, and shortage balance. Constraint (1.14) ensures that production quantities are equal to the satisfied demand. Constraint (1.15) shows the demand range. The next constraint shows the relation between setup and production feasibility. Constraint (1.17) ensures that the machine usage for production and setup has not exceeded the available capacity. The right side of this constraint estimates the available capacity. The numerator calculates the expected value of capacity. $P_r^{\tau r}\mu$ modifies the error of unequal. The denominator ensures that the value of τr is properly chosen. Constraint (1.18) shows that only one product can be produced in each microperiod. Constraint (1.19) ensures that if two different products are manufactured in two consecutive microperiods, then setup will be necessary. Constraint (1.20) counts the number of periods since the last PM. As long as PM is not performed, that is, $q_k^{nr} = 1$, the value of tr_{nrk} per each maintenance microperiod is one unit greater than that of the previous maintenance microperiod; otherwise, its value is only 1, which means that PM occurred in the maintenance microperiod nr. Constraint (1.23) also limits the minimum batch size production. The constraint is added for considering the minimum batch size (m_j), in each machine setup. It can be written as follows:

$$Q_{jnk} \geq m_j \times (Y_{jnk} - Y_{j(n-1)k}) \quad \forall j, n, k \tag{1.23}$$

In some industries, if setup occurs, the production batch size must then be greater than a minimum level due to technological or economic factors. This constraint ensures that the minimum batch size is produced after each setup. It ensures that if the setup for a product was not carried out in microperiod (position) $n-1$ but that it was in position n, then the product batch size in period t must be at least equal to the minimum batch size of the product.

1.3.5 Model Output Representation

The output of model should contain the production schedule and the PM schedule. The maximum number of lots equals the number of positions in the model. Therefore, the number of lots may be less than the number of positions. In this state, setup carryover is applied to the remaining positions at the end of the period. In other words, one setup state in each position is determined in the yield solution while production may plausibly not occur in some positions at the end of the period. The following conditions may be regarded as an example.

There are three types of products and five microperiods in each period. This means five lots can be produced in each period. If it is assumed that the triangular inequality conditions do not hold between setup times, there might be two or more product lots in one period. However, assuming that triangular conditions hold between setup times, production of a product occurs only in one lot in each period. Therefore, it will not be necessary for the number of microperiods in each period to exceed the number of product types. In addition, it is worth mentioning that if there are three types of products, there will be no need for the number of microperiods to be greater than the number of products under any assumption. For this problem size, there is no need for creating a complex state with more microperiods. However, as the number of products increases, problem complexity may also increase to the extent that prediction of the number of microperiods becomes impossible, especially when we are simultaneously faced with setup time and setup cost.

1.3.6 Hybrid Solution Algorithm

Given the fact that the PGLSP is NP-hard, the model presented in this chapter is NP-hard, too. Hence, efficient methods need to be developed that can obtain near-optimal solutions in a reasonable time for large-size instances. A simulating annealing (SA) algorithm and a hybrid algorithm have been developed for this purpose. The hybrid algorithm combines a heuristic algorithm and SA. The heuristic algorithm has two parts. Part 1 satisfies the minimum product demand. Part 2 assigns available capacities to products with higher profits. SA determines the product sequence and the PM schedule. Lot-sizing and demand quantity are obtained by the heuristic algorithm. The solutions obtained from solving the mathematical models have been used to assess the quality of the algorithms. Numerical results show the efficiency of the developed hybrid algorithm.

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